Monetary Policy without Commitment*

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Abstract

This paper studies the implications of central bank credibility for long-run inflation and inflation dynamics. We introduce central bank lack of commitment into a standard non-linear New Keynesian economy with sticky-price monopolistically competitive firms. Inflation is driven by the interaction of lack of commitment and the economic environment. We show that long-run inflation increases following an unanticipated permanent increase in the labor wedge or decrease in the elasticity of substitution across varieties. In the transition, inflation overshoots and then gradually declines. Quantitatively, inflation overshooting is persistent, and the welfare loss from lack of commitment relative to inflation targeting is large.

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Key Words: Policy rules, Monetary Policy, Policy Objectives, Inflation Targeting, Rules vs. Discretion

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1 Introduction

Inflation across advanced economies in the aftermath of the COVID-19 pandemic rose to levels not seen since the early 1980s. This has brought about a resurgence of interest in the subject of central bank credibility to the maintenance of low and stable inflation. These recent developments additionally highlight the challenge in applying the most commonly used quantitative macroeconomic models—which assume exogenous central bank reaction functions and inflation targets—for understanding the post-pandemic environment.

In this paper, we study central bank credibility by developing a framework where policy is not exogenous but is instead dynamically chosen by a central bank that maximizes social welfare in every date. We focus on the implications of the central bank's inability to make ex-ante commitments. Our analysis builds on the seminal work of Barro and Gordon (1983) and Rogoff (1985), who examine how the central bank's inflation-output tradeoff affects policy and the economy. This work and most of the vast literature that followed it, however, consider simple static settings or linearized dynamic environments.[1] By their construction, these analyses do not inform how central bank credibility impacts long-run inflation and transition dynamics.[2]

We introduce central bank lack of commitment into a standard New Keynesian model.[3] In order to allow for an analysis of long-run inflation and transition dynamics, we do not perform a linearization around a zero-inflation steady state but instead examine the fully non-linear model. For tractability, we take a deterministic environment and consider the impact of permanent shocks. The

[1] See, for example, Backus and Driffill (1985), Canzoneri (1985), Cukierman and Meltzer (1986), Athey, Atkeson, and Kehoe (2005), and Halac and Yared (2020, 2022).

[2] Linearized environments are useful for considering transition dynamics around an assumed steady state; however, such a state may not coincide with the actual steady state of the economy. In fact, the equilibrium under central bank lack of commitment in a linearized environment (e.g., Halac and Yared, 2022) will in general not coincide with the linearization of the equilibrium under lack of commitment in a non-linear environment. A full exposition of the non-linear game-theoretic environment is thus required to determine long-run outcomes.

[3] See Clarida, Galí, and Gertler (1999), Woodford (2003), and Galí (2015). As we describe next, we consider price stickiness with Calvo pricing (Calvo, 1983), but our results also hold under wage stickiness.

economy is composed of monopolistically competitive firms with sticky prices: in every period, a random fraction of firms have the ability to flexibly choose their price, while the remaining firms must keep their previous period's price. Wages are fully flexible, and households make consumption, labor, and savings decisions. Firms and households optimize taking into account current economic conditions and policies and their expectations of future economic conditions and policies. As is common in the literature, we allow for an exogenous labor wedge—a proportional positive or negative tax on labor—which captures statutory taxes and other labor market distortions, such as the pervasiveness of regulation and unionization.

Our economy admits two types of distortions. First, the existence of monopoly power means that absent a sufficiently negative labor wedge, firms underproduce and underhire. To examine the role of this distortion, we assume the labor wedge is large enough that underproduction arises under flexible prices. Second, the existence of sticky prices generates price dispersion in the goods market (if inflation is non-zero), which causes labor misallocation, with too much labor drawn to the production of low-price varieties and too little to the production of high-price varieties. Our analysis highlights how monopoly distortions and labor misallocation impact the central bank's inflation-output tradeoff and guide the conduct of monetary policy.

An important feature of our environment is that monetary policy is (neutral but) not superneutral in the long run. Different policy paths can lead to a continuum of potential steady states. Comparing across them, we find that steady states with higher inflation admit relatively lower monopoly distortions, as they generate more overhiring by sticky-price firms. At the same time, steady states with higher inflation admit relatively higher price dispersion and labor misallocation, as they generate a larger divergence in prices between flexible-price firms and sticky-price firms. These effects of inflation are muted if the central bank can commit to an optimal policy path: we show any steady state features zero inflation in that benchmark case. Thus, the economic environment has no long-run inflation implications under central bank commitment.

We study central bank lack of commitment by analyzing the Markov Perfect

Competitive Equilibria of our model (where central bank strategies and private sector beliefs depend only on payoff-relevant variables). In every period, flexible-price firms set prices, the central bank sets the interest rate, and markets open and clear. The central bank has discretion and freely chooses the interest rate that maximizes social welfare at each date, taking as given the distribution of prices. Firms choosing prices anticipate that the central bank has discretion today and in the future when forming expectations about future policy.

We show that an equilibrium is characterized by two difference equations. The dynamic path of inflation is given by an equation that is forward-looking, i.e. a non-linear Phillips curve where current inflation is a function of expectations of future inflation. The dynamic path of price dispersion is given by an equation that is backward-looking, i.e. where current dispersion is a function of past dispersion. These equations yield a unique steady state, thus allowing us to analyze transition dynamics around it. The tractability of the solution owes partly to the timing in our model, where, in each period, prices are set prior to the choice of interest rates. Because the central bank takes as predetermined the distribution of prices today and thus also the continuation equilibrium tomorrow (as the equilibrium is Markov), it optimally chooses an interest rate that maximizes static welfare conditional on the level of price dispersion.[4] The result is a policy that eliminates monopoly distortions and sets the labor share to 1,[5] and an equilibrium that can be simplified to a system of two equations.

Our main results are as follows. First, we show that long-run inflation is determined by the interaction of the central bank's lack of commitment and the economic environment. Specifically, long-run inflation is higher the higher the labor wedge and the lower the elasticity of substitution across varieties. To understand this result, consider first the incentives of the central bank to deviate from an equilibrium policy by cutting interest rates. A rate cut increases

[4] Our results are unchanged if firms and the central bank move simultaneously, as in Barro and Gordon (1983). However, if the central bank moves before firms set prices, then off-path expectations by firms can play a role in equilibrium selection, complicating the analysis.

[5] In other words, average firm profits (net of labor taxes) across the economy equal zero, with some sticky-price firms making negative profits and all flexible-price firms making positive profits.

consumption at the cost of increased labor effort, so its marginal benefit is increasing in monopoly distortions (which suppress labor) and decreasing in price dispersion and labor misallocation (which reduce aggregate productivity).

Starting from a given steady state, suppose there is a permanent increase in the labor wedge or a permanent decrease in the elasticity of substitution across varieties (with these changes being unanticipated). A central bank with commitment can respond by preserving inflation stability. However, a central bank without commitment has an incentive to undo the resulting increase in monopoly distortions by cutting interest rates and stimulating output. Flexible-price firms anticipate this and rationally forecast higher future labor demand and real wages (relative to the commitment case), which necessitate higher offsetting prices today. Hence, over time, flexible-price firms increase prices, leading to higher price dispersion and lower aggregate productivity. The economy converges to a new steady state once aggregate productivity declines sufficiently that the central bank no longer benefits from cutting interest rates. In that new steady state, inflation and price dispersion are both higher.

Our second main result characterizes the transition as the economy moves from an initial steady state to one with higher inflation. We show this transition features inflation overshooting. Starting from a given steady state, consider a permanent increase in the labor wedge or a permanent decrease in the elasticity of substitution across varieties. As just described, price dispersion rises as the economy moves towards a higher-inflation steady state. The central bank sees a relatively larger benefit to stimulating output early in the transition when price dispersion and labor misallocation are low; as they rise, it becomes less worthwhile to increase labor to generate additional consumption. Flexible-price firms realize that monetary stimulus will be larger earlier in the transition, so they offset the ensuing higher wage costs with price increases that are also larger earlier on. The implication of these dynamics is inflation overshooting.

Our final main result explores the quantitative implications of our analysis. Using standard parameterizations of the New Keynesian model, we evaluate the response of the economy to a permanent increase in the labor wedge or a permanent decrease in the elasticity of substitution across varieties. In both cases, inflation jumps up following the shock and then gradually declines towards a new higher steady-state level. Nominal interest rates jump up and gradually increase to a higher level,[6] while output falls gradually as price dispersion and labor misallocation increase in the transition. We find that inflation overshooting is persistent. Furthermore, compared to an economy with commitment to inflation targeting, the welfare loss from lack of commitment is quantitatively large. The large magnitudes implied by our model owe to the fact that firms are highly forward-looking in the New Keynesian framework, which means that the steady-state labor share is relatively insensitive to inflation.

Our results show how exogenous economic factors can interact with central banks' lack of commitment and impact long-run inflation and inflation dynamics. In Afrouzi, Halac, Rogoff, and Yared (2024), we apply a greatly simplified version of the model of this paper to shed light on the forces that drove global inflation downward over the four decades prior to the COVID-19 pandemic, including globalization, the Washington consensus, and deunionization. We further use the analysis to argue that several economic trends of the post-pandemic period—deglobalization, fiscal pressures, and rising long-term real interest rates—will likely increase central banks' incentives to inflate. Absent reforms, periods of high inflation could thus become more common in the coming decade compared to the past.

Related Literature. Our paper fits into the literature on central bank credibility and reputation pioneered by Barro and Gordon (1983) and Rogoff (1985).[7] As discussed, we depart from this literature by analyzing the equilibrium of a fully non-linear New Keynesian model. This departure allows us to examine the endogenous dynamic evolution of the central bank's inflation-output tradeoff as well as the quantitative implications of central bank credibility. An approach that considers a Markov equilibrium under lack of commitment around a linearized (distorted) zero-inflation, zero-dispersion steady state (as in Halac and Yared, 2022, for example) not only features no transition dynamics but also

[6]The higher steady-state level for nominal interest rates reflects the Fisherian effect, which is present in the non-linear New Keynesian model.

[7]See additionally the work cited in Footnote 1.

significantly overestimates the effect of permanent shocks on long-run inflation relative to the non-linear model.[8]

Previous work has studied lack of commitment to monetary policy in non-linear environments. For example, many models of fiscal policy are concerned with the central bank's commitment to not inflating away public debt (e.g., Alvarez, Kehoe, and Neumeyer, 2004; Aguiar, Amador, Farhi, and Gopinath, 2015). Dávila and Schaab (2023) show that lack of commitment to monetary policy has distributional implications in heterogeneous-agent economies.[9] We depart from this literature by considering the cost of price dispersion that results from price stickiness in standard New Keynesian models, and by examining how this cost dynamically affects the central bank's inflation-output tradeoff.

A related literature studies the inflation-output tradeoff under lack of commitment in non-linear settings. The focus of this literature has been on identifying conditions for equilibrium multiplicity (see, e.g., Albanesi, Chari, and Christiano, 2003; King and Wolman, 2004; Zandweghe and Wolman, 2019). These considerations do not arise in our setting, where we obtain a unique equilibrium. We depart from this work by providing an analytical characterization of the unique steady state and the transition dynamics, and by analytically studying how these depend on the economic environment.[10]

By considering a benchmark setting with full central bank commitment, we relate to prior work on the optimal commitment policy in the non-linear New Keynesian model. This literature has shown that a zero-inflation steady state exists under commitment (see Benigno and Woodford, 2005; Yun, 2005; Schmitt-Grohé and Uribe, 2011). We expand their results by showing that any steady state must have zero inflation in our commitment benchmark.

[8] The reason is that the linearized model underestimates the welfare costs of rising price dispersion due to inflation.

[9] Their economy has sticky prices with exogenous costs of price adjustment (as opposed to Calvo pricing in our model); as a consequence, their equilibrium has no price dispersion.

[10] Prices are sticky only within the period in Albanesi, Chari, and Christiano (2003) and only across two periods in King and Wolman (2004). Eggertsson and Swanson (2008) obtain a unique equilibrium in a version of King and Wolman (2004) that takes the central bank and private sector to move simultaneously. The model of Zandweghe and Wolman (2019) is the closest to ours with Calvo pricing across periods, but their timing is different and their results under lack of commitment are numerical rather than analytical.

Finally, our paper makes a methodological contribution by providing a novel recursive representation of the non-linear Phillips curve. We do so by defining an auxiliary variable that captures the passthrough of real wages to current inflation, holding future inflation expectations fixed. The recursive representation allows us to characterize transition dynamics, and we conjecture it could be useful in future analyses of the non-linear New Keynesian model.[11]

2 Model

We study a standard non-linear New Keynesian model (Clarida, Galí, and Gertler, 1999; Woodford, 2003; Galí, 2015). There is a unit mass of monopolistically competitive firms that set prices under Calvo-style rigidity (Calvo, 1983). Wages are fully flexible,[12] and households make consumption, labor, and savings decisions. Monetary policy is conducted by a central bank that lacks commitment and freely chooses the interest rate that maximizes social welfare at each date. Firms and households optimize taking into account current economic conditions and policies and their expectations of future economic conditions and policies, given the central bank's lack of commitment.

Households. At every date $t \in \{0, 1, 2, \dots\}$, the representative household chooses its consumption $C_{j,t}$ of each firm variety $j \in [0, 1]$, its labor supply L_t, its holdings B_t of a risk-free nominal government bond that pays interest i_t, and its holdings $s_{j,t}$ of shares of each firm $j \in [0, 1]$. Denote firm j's variety price by $P_{j,t} > 0$, its nominal share price by $P^S_{j,t}$, and its nominal profits by $X_{j,t}$. Letting W_t denote the nominal wage, the household's problem is

$$\max_{C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j \in [0,1]}} \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{L_t^{1+\psi}}{1+\psi} \right) \tag{1}$$

subject to

[11] In particular, this representation may be useful in other work studying dynamics away from the zero-inflation benchmark, as in Ascari (2004) and Ascari and Sbordone (2014).

[12] Our analysis can be analogously applied to a setting with sticky wages and flexible prices.

$$\int_0^1 P_{j,t}C_{j,t}\mathrm{d}j + B_t \leq W_t L_t + (1+i_{t-1})B_{t-1} + \int_0^1 s_{j,t}X_{j,t}\mathrm{d}j + \int_0^1 (s_{j,t-1} - s_{j,t})P^S_{j,t}\mathrm{d}j - T_t,$$

$$\text{where } C_t = \left(\int_0^1 C_{j,t}^{1-\sigma^{-1}}\mathrm{d}j\right)^{\frac{1}{1-\sigma^{-1}}}.$$

C_t denotes the aggregate consumption bundle and T_t is a lump sum tax. We have taken $\beta \in (0,1)$ to be the discount factor, $\sigma > 1$ the elasticity of substitution across varieties, and $\psi > 1$ the inverse elasticity of labor supply.

The household's optimization yields that the demand for variety j satisfies

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma}, \tag{2}$$

where $P_t = \left(\int_0^1 P_{j,t}^{1-\sigma}\mathrm{d}j\right)^{\frac{1}{1-\sigma}}$. Thus, we can write $\int_0^1 P_{j,t}C_{j,t}\mathrm{d}j = P_t C_t$ in the budget constraint in program (1), which means that households can choose their consumption bundle C_t as a function of P_t before solving the subproblem of choosing the consumption $C_{j,t}$ of each variety $j \in [0,1]$ as a function of $(P_{j,t})_{j\in[0,1]}$. The intratemporal condition is

$$\frac{W_t}{P_t} = C_t L_t^{\psi}, \tag{3}$$

and the intertemporal condition is

$$1 = \beta(1+i_t)\frac{P_t C_t}{P_{t+1}C_{t+1}}. \tag{4}$$

The transversality condition requires that for each date t and firm j,[13]

$$\lim_{h\to\infty} \frac{1}{\prod_{\ell=0}^{h}(1+i_{t+\ell})}\mathbb{E}_t^j[X_{j,t+1+h}] = \lim_{h\to\infty} \frac{1}{\prod_{\ell=0}^{h}(1+i_{t+\ell})} i_{t+h}B_{t+h} = 0. \tag{5}$$

The expectation $\mathbb{E}_t^j[\cdot]$ operates over firm j's future idiosyncratic shocks, which we

[13] This condition combines a household optimality condition and a no-Ponzi condition in a complete market environment that allows for Arrow-Debreu securities, including securities that pay off at a future date an amount proportional to the profits of any given firm conditional on any given history.

discuss subsequently. No arbitrage for stocks requires $P^S_{j,t} = X_{j,t} + \mathbb{E}^j_t[P^S_{j,t+1}]/(1+i_t)$, which combined with (5) yields

$$P^S_{j,t} = X_{j,t} + \sum_{h=0}^{\infty} \frac{1}{\prod_{\ell=0}^{h}(1+i_{t+\ell})} \mathbb{E}^j_t[X_{j,t+1+h}]. \tag{6}$$

Combining (6) with the intertemporal condition for arbitrary horizon $t+h$, we obtain that the nominal share price of firm j satisfies

$$P^S_{j,t} = \sum_{h=0}^{\infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \mathbb{E}^j_t[X_{j,t+h}]. \tag{7}$$

Firms. Each firm $j \in [0,1]$ produces with technology $Y_{j,t} = L_{j,t}$. Given the labor $L_{j,t}$ demanded by each j, we have $L_t = \int_0^1 L_{j,t} \mathrm{d}j$ in every period t.

Firms set prices as in Calvo (1983). In every period, a random fraction $1-\theta \in (0,1)$ of firms are able to flexibly choose their prices; the remaining fraction θ must keep their previous period's price. We let $(P_{j,-1})_{j\in[0,1]}$ be the exogenous initial distribution of prices. Firms commit to producing enough to meet demand given their price $P_{j,t}$, even if that means making negative profits.

Firms are subject to a proportional payroll tax $\tau \in (0,1)$, which we refer to as the labor wedge. This wedge captures statutory taxes on labor and other labor market distortions, such as the pervasiveness of regulation and unionization.[14] We make the following assumption:

Assumption 1. *The labor wedge satisfies $\tau > -1/\sigma$.*

Assumption 1 implies that monopoly distortions arise in an economy with fully flexible prices. As we will see, it will also guarantee that monopoly distortions are present in the steady state of our economy.

Firm profits at any date t satisfy

$$X_{j,t} = P_{j,t} Y_{j,t} - (1+\tau) W_t L_{j,t},$$

[14] Under this latter interpretation, $-T_t$ can be thought of as measuring union profits.

which, combined with (7), implies

$$P_{j,t}^{S}=\sum_{h=0}^{\infty}\beta^{h}\frac{P_{t}C_{t}}{P_{t+h}C_{t+h}}\mathbb{E}_{t}^{j}\left[P_{j,t+h}Y_{j,t+h}-\left(1+\tau\right)W_{t+h}L_{j,t+h}\right]. \tag{8}$$

A firm choosing its variety price $P_{j,t}$ at date t maximizes its share price $P_{j,t}^{S}$ given by (8), taking into account that $P_{j,t}$ will prevail at date $t+h$ with probability θ^{h}. The flexible-price firm's problem thus yields a price P_{t}^{*} that solves

$$\max_{P_{t}^{*}}\sum_{h=0}^{\infty}(\beta\theta)^{h}\frac{P_{t}C_{t}}{P_{t+h}C_{t+h}}[P_{t}^{*}-(1+\tau)W_{t+h}]C_{t+h}\left(\frac{P_{t}^{*}}{P_{t+h}}\right)^{-\sigma}, \tag{9}$$

where we have taken into account that the transversality condition implies, for all t,

$$\lim_{h\to\infty}(\beta\theta)^{h}\frac{P_{t}C_{t}}{P_{t+h}C_{t+h}}[P_{t}^{*}-(1+\tau)W_{t+h}]C_{t+h}\left(\frac{P_{t}^{*}}{P_{t+h}}\right)^{-\sigma}=0. \tag{10}$$

Government. At every date t, the central bank sets the interest rate i_{t} to maximize social welfare, which is given by (1). We postpone the description of the central bank's problem until Section 4.

The fiscal authority sets taxes T_{t} and debt B_{t} to satisfy its budget constraint:

$$(1+i_{t-1})B_{t-1}=B_{t}+T_{t}+\tau W_{t}L_{t}. \tag{11}$$

We let B_{-1} be the exogenous initial level of government debt.

Order of Events. The order of events at any given date t, given the previous period's price distribution $(P_{j,t-1})_{j\in[0,1]}$, is as follows:

1. Flexible-price firms choose price $P_{j,t}=P_{t}^{*}$; sticky-price firms set $P_{j,t}=P_{j,t-1}$.
2. The central bank chooses monetary policy, i.e. the interest rate i_{t}.
3. Households choose consumption, labor, and savings $C_{t},L_{t},B_{t},(s_{i,t},C_{j,t})_{j\in[0,1]}$.
4. The fiscal authority chooses fiscal policy, i.e. taxes T_{t} and debt B_{t}.

Observe that monetary policy is chosen after firms have chosen their prices.

Hence, if the central bank deviates from an equilibrium policy, firms are no longer optimizing during the period of the deviation off path.[15] The fact that fiscal policy is chosen at the end of the period is for expositional simplicity and without loss given our equilibrium definition that we describe next.

Equilibrium Definition. Our solution concept is Markov Perfect Competitive Equilibrium (MPCE), in which households, firms, and the government make decisions as a function of payoff-relevant variables only. Note that given the presence of lump sum taxes, our economy features Ricardian Equivalence, so the level of debt B_t can be treated as payoff irrelevant. As such, we assume that at the end of each period, the fiscal authority chooses debt $B_t = 0$ and sets taxes T_t to balance its budget, both on and off the equilibrium path.[16]

Formally, let Ω_{t-1} represent the distribution of prices across firms at date $t-1$. Conditional on Ω_{t-1}, flexible-price firms choose prices and sticky-price firms keep their previous period's price, determining Ω_t. Let Γ denote the corresponding mapping, with $\Omega_t \equiv \Gamma\left(\Omega_{t-1}\right)$. Given Ω_t, the central bank chooses monetary policy $i_t \equiv \Psi(\Omega_t)$, where Ψ is the central bank's reaction function. Finally, given Ω_t and i_t, households choose consumption, labor, and savings $\left(C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j\in[0,1]}\right) \equiv \omega(\Omega_t, i_t)$, where ω is the households' reaction function. In equilibrium, $B_t = 0$ and $s_{j,t} = 1$, since households can be treated identically without loss of generality.

An MPCE is a collection of mappings (Γ, Ψ, ω) such that, at every date t and given (Γ, Ψ, ω), the mapping $\Gamma(\Omega_{t-1})$ satisfies flexible-price firm optimality, $\Psi(\Omega_t)$ maximizes social welfare, and $\omega(\Omega_t, i_t)$ satisfies household optimality.[17]

[15] As noted in the Introduction, our results are unchanged if firms and the central bank move simultaneously.

[16] The set of continuation MPCE at a date t starting from any two values of debt is the same. Therefore, the fiscal authority at the end of the period is indifferent over all values of B_t and can set debt to zero. Without the Markov restriction, government debt could serve as a payoff-irrelevant coordination device to select among different equilibria.

[17] Observe an MPCE is a sustainable equilibrium, as defined in Chari and Kehoe (1990).

3 Competitive Equilibrium

Any MPCE is a competitive equilibrium, i.e., it satisfies firm and household optimality given the central bank's policy. In Section 3.1, we characterize the conditions for a sequence of aggregate allocations and prices to constitute a competitive equilibrium given a sequence of policies. We use these conditions in Section 3.2 to illustrate the non-superneutrality of monetary policy and how long-run inflation is related to price dispersion and monopoly distortions in a hypothetical steady state. In Section 3.3, we characterize the steady state of the economy in a benchmark setting with central bank commitment.

3.1 Equilibrium Conditions

To describe the necessary and sufficient conditions for a competitive equilibrium, we first define the labor share and derive the dynamics of price dispersion, the Phillips curve, and a transversality condition.

Aggregate Production. Define price dispersion $D_t \geq 1$ by[18]

$$D_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma} \mathrm{d}j.$$

Observe that $C_{j,t} = Y_{j,t} = L_{j,t}$, where

$$C_t = Y_t = \left(\int_0^1 Y_{j,t}^{1-\sigma^{-1}} \mathrm{d}j\right)^{\frac{1}{1-\sigma^{-1}}}. \tag{12}$$

[18] We can show that $D_t \geq 1$, with equality only when all prices are almost surely equal. To see this, define the function $f(X) \equiv X^{\frac{\sigma}{\sigma-1}}$ and note that $D_t = \mathbb{E}_j[f((P_{j,t}/P_t)^{1-\sigma})]$, where the expectation is taken according to the Lebesgue measure over $j \in [0,1]$. Now note that $f(\cdot)$ is strictly convex for $\sigma > 1$ and thus, by Jensen's inequality, we have $\mathbb{E}_j[f((P_{j,t}/P_t)^{1-\sigma})] > f(\mathbb{E}_j[(P_{j,t}/P_t)^{1-\sigma}])$, with equality when $P_{j,t}/P_t = 1$ almost surely with respect to the Lebesgue measure. Finally, note that by definition of the aggregate price, $\mathbb{E}_j[(P_{j,t}/P_t)^{1-\sigma}] = 1$, so that $\mathbb{E}_j[f((P_{j,t}/P_t)^{1-\sigma})] > f(1) = 1$.

Thus, we can write

$$L_t = \int_0^1 L_{j,t} \mathrm{d}j = \int_0^1 C_{j,t} \mathrm{d}j = \int_0^1 C_t \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} \mathrm{d}j = C_t D_t,$$

which, given $C_t = Y_t$, implies

$$Y_t = \frac{L_t}{D_t}. \tag{13}$$

This relationship shows that conditional on a level of labor L_t, higher price dispersion D_t reduces aggregate production Y_t and thus aggregate consumption C_t. The reason is that households spend too much on low-price varieties and too little on high-price varieties, so too much labor is drawn to the production of low-price varieties and too little to the production of high-price varieties.

Using $C_t = Y_t$, we can rewrite the Euler equation (4) as

$$1 = \beta \frac{(1 + i_t)}{\Pi_{t+1}} \frac{Y_t}{Y_{t+1}}, \tag{14}$$

where Π_{t+1} is the gross level of inflation:

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t}. \tag{15}$$

Using (13), we can rewrite the intratemporal condition (3) as

$$\frac{W_t}{P_t} = D_t^{\psi} Y_t^{1+\psi}. \tag{16}$$

This relationship shows that the real wage increases with output and with price dispersion. The reason for the latter is that the higher is price dispersion, the more households end up overworking to produce low-price varieties.

To facilitate future discussion, we define the labor share μ_t by

$$\mu_t = \frac{W_t L_t}{P_t Y_t}.$$

The labor share is inversely related to monopoly profits and therefore captures the extent of monopoly distortions. Using (13) and (16), we obtain

$$\mu_t = (D_t Y_t)^{1+\psi}. \tag{17}$$

Holding output fixed, greater price dispersion results in higher real wages (to induce overworking on low-price varieties), thus increasing the labor share. Moreover, holding price dispersion fixed, higher output results in higher real wages and higher labor, thus also increasing the labor share.

Dispersion Dynamics. Since a random fraction $1-\theta$ of firms are able to adjust their prices in every period, the price at time t satisfies

$$P_t^{1-\sigma} = (1-\theta)(P_t^*)^{1-\sigma} + \theta P_{t-1}^{1-\sigma}.$$

Using the definition of gross inflation in (15) and rearranging terms yields

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}. \tag{18}$$

Intuitively, this relationship says that the larger is the upward price adjustment from P_t to P_t^*, the higher is the level of inflation Π_t.

The dynamics of price dispersion are given by

$$D_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\sigma} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{-\sigma} D_{t-1},$$

or equivalently, substituting with (15) and (18),

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1}. \tag{19}$$

The initial level D_{-1} is given by the initial price distribution $(P_{j,-1})_{j\in[0,1]}$.

The relationship in (19) is backward looking, with dispersion at t being a positive function of dispersion at $t-1$. As for the effect of inflation on dispersion, (19) shows there are two forces at play. On the one hand, higher inflation causes sticky-price firms to be left further behind, which raises dispersion (second term on the right-hand side). On the other hand, higher inflation causes flexible-price firms catch up to a higher price level, which reduces dispersion (first term on the right-hand side). One can show that for non-negative inflation ($\Pi_t \geq 1$), the first force dominates, so higher inflation leads to higher price dispersion.

Phillips Curve. The first-order conditions from the flexible-price firm's problem in (9) yield that at each date t,

$$\frac{P_t^*}{P_t} = \frac{\sigma}{\sigma - 1} \frac{\sum_{h=0}^{\infty} (\beta\theta)^h \left(\frac{P_{t+h}}{P_t}\right)^{\sigma} \frac{(1+\tau)W_{t+h}}{P_{t+h}}}{\sum_{h=0}^{\infty} (\beta\theta)^h \left(\frac{P_{t+h}}{P_t}\right)^{\sigma-1}}.$$

We introduce an auxiliary variable δ_t defined by

$$\delta_t^{-1} \equiv \sum_{h=0}^{\infty} (\beta\theta)^h \left(\frac{P_{t+h}}{P_t}\right)^{\sigma-1},$$

which, using (15), can be rewritten recursively as

$$\delta_t^{-1} = 1 + \beta\theta \Pi_{t+1}^{\sigma-1} \delta_{t+1}^{-1}. \tag{20}$$

This variable allows us to significantly simplify the analysis. Substituting with δ_t^{-1} and (16), the first-order conditions above can be rewritten as

$$\frac{P_t^*}{P_t} = \frac{\sigma}{\sigma - 1} \delta_t \sum_{h=0}^{\infty} (\beta\theta)^h \left(\frac{P_{t+h}}{P_t}\right)^{\sigma} (1+\tau) D_{t+h}^{\psi} Y_{t+h}^{1+\psi},$$

or, recursively,

$$\frac{P_t^*}{P_t} = \frac{\sigma(1+\tau)}{\sigma - 1} \delta_t D_t^{\psi} Y_t^{1+\psi} + \beta\theta \frac{\delta_t}{\delta_{t+1}} \Pi_{t+1}^{\sigma} \frac{P_{t+1}^*}{P_{t+1}}.$$

Further substituting with (15), (18), and (20) yields a non-linear Phillips curve:

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^{\psi} Y_t^{1+\psi} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}. \tag{21}$$

The relationship in (21) is forward looking, with current inflation being a positive function of expectations of future inflation. Specifically, since flexible-price firms take into account the path of current and future marginal costs when adjusting their prices, inflation today is increasing in the expectation of real wages today (given by $D_t^{\psi} Y_t^{1+\psi}$) and of future inflation. Observe that δ_t, which captures the sensitivity of current inflation to current real wages, has a useful interpretation of being related to the slope of the Phillips curve.

Transversality Condition. Combining equation (10) together with (15), (16), and (18), and noting that $P_t C_t \left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} > 0$, we can rewrite the transversality condition to require, for each date t,

$$\lim_{h\to\infty}\left[\beta\theta\left(\prod_{\ell=1}^{h}\Pi_{t+\ell}\right)^{\frac{\sigma}{h}}\right]^{h}\left[\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}\frac{1}{\prod_{\ell=1}^{h}\Pi_{t+\ell}} - (1+\tau)Y_{t+h}^{1+\psi}D_{t+h}^{\psi}\right] = 0. \tag{22}$$

Observe that if inflation converges and $\lim_{h\to\infty}\Pi_{t+h} \geq 1$, then this condition can be satisfied in the long run only if $\lim_{h\to\infty}\Pi_{t+h} < (\beta\theta)^{-1/\sigma}$.

Necessary and Sufficient Conditions. Our analysis thus far leads to a system of equations that must necessarily hold in each period t in a competitive equilibrium. The next lemma shows that these conditions are not only necessary but also sufficient for the construction of a competitive equilibrium.

Lemma 1. *Given an initial price distribution $(P_{j,-1})_{j\in[0,1]}$ and a sequence of policies $(i_t)_{t=0}^{\infty}$, a sequence of allocations and prices $(L_t, Y_t, D_t, \delta_t, \Pi_t)_{t=0}^{\infty}$ is supported by a competitive equilibrium if and only if it satisfies conditions* (13), (14), (19), (20), (21), *and* (22).

An implication of the lemma is that price dispersion D_{t-1} is a sufficient statistic for the distribution of prices Ω_{t-1}. That is, the set of continuation MPCE at a date t starting from any two price distributions with the same dispersion is the same. We can thus refine our Markov restriction by replacing Ω_{t-1} with D_{t-1} in the strategies of households, firms, and the central bank.[19]

3.2 Long-Run Monetary Non-Superneutrality

An important feature of our environment is that monetary policy is not superneutral in the long run.[20] Define a steady state as finite and constant values for $L_t, Y_t, D_t, \delta_t, \Pi_t$ under a constant policy i_t, with D_t converging to a finite steady-state value independent of its initial value. Using the dispersion dynamics equation (19), the latter requires $\Pi < \theta^{-1/\sigma}$, as otherwise D_t would diverge to infinity. Equations (19)-(21) can be combined to yield the following steady-state conditions:

$$D = \frac{1-\theta\Pi^{\sigma-1}}{1-\theta\Pi^{\sigma}} \left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{1}{\sigma-1}}, \tag{23}$$

$$\frac{\mu}{D^{1+\psi}} = \frac{\sigma-1}{\sigma(1+\tau)} \left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{1+\psi}{1-\sigma}} \left(\frac{1-\theta\Pi^{\sigma}}{1-\theta\Pi^{\sigma-1}}\right)^{\psi} \frac{1-\beta\theta\Pi^{\sigma}}{1-\beta\theta\Pi^{\sigma-1}}, \tag{24}$$

where we have used that the steady-state labor share satisfies $\mu = (DY)^{1+\psi}$.

The next lemma considers steady states satisfying $\Pi \geq 1$, as this will be the relevant case when we study equilibrium policy in the next section.

Lemma 2. *Given a fixed gross inflation level $\Pi \geq 1$, there are unique values (D, μ) of price dispersion and labor share that satisfy the steady-state conditions (23)-(24). Moreover, D and μ are both strictly increasing in Π.*

Higher inflation leads to a larger divergence in prices between flexible-price firms and sticky-price firms. Thus, steady states with higher inflation admit

[19] Without the Markov restriction, the distribution of prices could serve as a payoff-irrelevant coordination device to select among different equilibria.

[20] King and Wolman (1996) discuss the non-superneutrality of money under Calvo-style price rigidity. This feature is consistent with empirical evidence; see, e.g., Ascari, Bonomolo, and Haque (2024).

higher price dispersion and labor misallocation. At the same time, higher inflation means that sticky-price firms overproduce and overhire more. Thus, steady states with higher inflation also admit a higher labor share and lower monopoly distortions. We note that there is a second, opposing force that pushes the labor share down, as flexible-price firms increase their prices by more under higher inflation (to protect against the possibility of overhiring in the future if unable to change prices). However, because of discounting, this second force is dominated by the overhiring force from sticky-price firms.

Two observations about potential steady states are useful to keep in mind for our analysis in the next sections. First, Assumption 1 implies that monopoly distortions are present in a zero-inflation, zero-dispersion steady state (by (23)-(24), if $\Pi = 1$, then $D = 1$ and $\mu = (\sigma - 1)/[\sigma(1 + \tau)] < 1$). Hence, there is a tension between reducing price dispersion and reducing monopoly distortions; this tension will guide the central bank's dynamic inflation-output tradeoff.

Second, our environment does not pin down transition dynamics for inflation. Specifically, consider a hypothetical transition from an initial steady state with inflation Π to one with inflation $\Pi' > \Pi$. There are multiple potential transition paths between the steady states that are consistent with the conditions in Lemma 1. One potential transition path has inflation immediately jumping from Π to Π', with D_t and μ_t (through Y_t) evolving according to (19)-(21). Other transition paths may admit both temporary and permanent changes in inflation. The implication is that economic forces by themselves do not determine inflation dynamics. Any inflation dynamics that emerge in our model are driven by the interaction of economic forces with the central bank's policy.

3.3 Full Commitment Benchmark

As a benchmark for our analysis, we study the policy the central bank would choose at date 0 under full commitment. Using (12) and (13) to rewrite household welfare in (1), the central bank's commitment problem is

$$\max_{Y_t, D_t, \Pi_t, \delta_t} \sum_{t=0}^{\infty} \beta^t \left(\log(Y_t) - \frac{(D_t Y_t)^{1+\psi}}{1+\psi} \right)$$

$$\text{subject to}$$
$$(19), (20), (21), \text{and } (22).$$

Prior studies, including Benigno and Woodford (2005), Yun (2005), and Schmitt-Grohé and Uribe (2011), find that a zero-inflation steady state exists in New Keynesian models with central bank commitment. We expand these results by showing that a zero-inflation steady state not only satisfies the optimality conditions of the commitment problem but is also the unique steady state that does so.

Proposition 1. *Under full central bank commitment, any steady state has zero inflation.*

On the one hand, by increasing inflation at a date t, the central bank can stimulate demand, which mitigates intratemporal distortions at t by increasing the labor share towards 1. On the other hand, increasing inflation at t also increases price dispersion and reduces aggregate productivity at all dates prior to t. The reason is that dispersion at t is a function of inflation at t (see (19)), which depends on current and future intratemporal distortions (see (21)). Thus, as $t \to \infty$, the benefit of reducing intratemporal distortions at t is outweighed by the costs of reducing aggregate productivity at all prior dates, implying that committing to zero long-run inflation is optimal.[21]

While zero long-run inflation is optimal from the perspective of date 0, it is not optimal from the perspective of the long run given Assumption 1. The zero-inflation steady state admits lower welfare than a steady state with some arbitrarily low inflation level, as the latter generates a first-order benefit by mitigating intratemporal distortions in exchange for a second-order loss from increasing price dispersion above zero. Similarly, it is not optimal to "jump" to

[21] This intuition is related to that in models where optimal long-run taxes on capital are zero (see Judd, 1985; Chamley, 1986; Atkeson, Chari, and Kehoe, 1999; Straub and Werning, 2020). Under positive inflation, flexible-price firms trade off positive profits in the short run against negative profits in the long run, and higher inflation is an intertemporal distortion that magnifies that tradeoff. In the long run, all firm pricing decisions are made in anticipation of future inflation, and it is not optimal to have permanent intertemporal distortions on firms, as these permanently reduce economic efficiency.

the zero-inflation steady state immediately at date 0, since there is a benefit of reducing distortions via inflation in the short run.[22] Commitment is thus key for the result in Proposition 1: the optimal policy at date 0 minimizes intratemporal distortions in the short-run by committing to an inflation path that converges to zero only in the long run.

An implication of Proposition 1 is that changes in the economic environment have no effect on long-run inflation under central bank commitment. In the next section, we examine how this conclusion changes under lack of commitment.

4 Equilibrium Policy

In this section, we characterize the policy chosen by the central bank without commitment at every date. We show how this policy determines the system of equations defining the steady state of the economy and transition dynamics.

Central Bank Problem. At every date t, the central bank sets the interest rate i_t to maximize social welfare given by (1). Given a price distribution Ω_t implying price dispersion D_t, and using (13), we can write social welfare at t recursively:

$$V(D_t) = \log(Y_t) - \frac{(D_t Y_t)^{1+\psi}}{1+\psi} + \beta V(D_{t+1}), \tag{25}$$

where we have taken into account that, by the Markov structure of the equilibrium, welfare depends only on the dispersion of prices.

Note that from the perspective of the central bank at t, the current price distribution Ω_t, which yields the price level P_t and price dispersion D_t, is predetermined. Furthermore, by the Markov structure, the price distribution at $t+1$ is also predetermined and is given by $\Omega_{t+1} = \Gamma(\Omega_t)$. This means that the central bank choosing interest rates at t cannot influence future price dispersion D_{t+1} or the value of $V(D_{t+1})$. Since it also takes P_{t+1} (and thus Π_{t+1}) and Y_{t+1} as given, the Euler equation (14) implies that the central bank can choose

[22] We emphasize the role of Assumption 1. As shown by Yun (2005), in an economy with $\tau = -1/\sigma$ and $D_{-1} = 1$, the optimal inflation rate is zero starting from date 0.

Y_t by choosing i_t without affecting future variables (off the equilibrium path). The derivative of the right-hand side of (25) with respect to Y_t thus reduces to

$$\frac{1}{Y_t} - D_t^{1+\psi} Y_t^{\psi}. \tag{26}$$

A rate cut by the central bank increases consumption (the first term in (26)) at the cost of increased labor effort (the second term in (26)). The marginal benefit of a rate cut is decreasing in price dispersion D_t, which reduces aggregate labor productivity by raising labor misallocation. Moreover, for $Y_t < D_t^{-1}$, the marginal benefit of a rate cut is higher the lower output Y_t, since a lower output level (caused by monopoly distortions) is associated with a larger gap between the marginal rate of substitution and the marginal product of labor.

Setting (26) to zero, the central bank's first-order condition yields

$$Y_t = D_t^{-1} \ . \tag{27}$$

The central bank chooses interest rates to undo all monopoly distortions and close the gap between the marginal rate of substitution and the marginal product of labor. By (17), the central bank thus sets the labor share μ_t to 1.

We make three remarks about the central bank's policy. First, the central bank does not internalize how firms' anticipation of its policy at t affects the prevailing price distribution Ω_t, which affects price dispersion D_t. This price distribution is determined by firm decisions made in all periods prior to t. This feature of our dynamic model captures the classic commitment problem addressed in the static models of Barro and Gordon (1983) and Rogoff (1985).[23]

Second, substituting the central bank's first-order condition (27) into the Euler equation yields a reaction function

$$1 + i_t = \frac{1}{\beta} \Pi_{t+1} Y_{t+1} D_t. \tag{28}$$

[23] Moreover, note that while (20) and (21) hold along the equilibrium path, they need not hold off the equilibrium path if the central bank deviates, since in that case firms would have chosen their prices without the correct anticipation of policy.

This reaction function—which emerges from dynamic optimization on and off the equilibrium path—shares several properties with the exogenous Taylor rules that are typically used to evaluate quantitative monetary models. In particular, the interest rate is increasing in future expected inflation and future expected output. It also reacts to the degree of price dispersion today; as noted above, holding future expectations fixed, higher price dispersion reduces aggregate productivity and thus the benefit of stimulating the economy. This will be important when we study the evolution of central bank incentives along the equilibrium path in the next section.

Finally, we observe that the central bank's policy of setting the labor share to 1 is independent of the underlying firm-price-setting model. The same policy would be optimal under lack of commitment in other environments with sticky prices, such as menu-cost or rational-inattention models.

System of Equations. An MPCE is characterized by combining the conditions in Lemma 1 (specifically (19)-(21)) with the central bank's first-order condition (27). This yields a system of two equations defining the dynamics of price dispersion D_t and inflation Π_t:

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1}, \tag{29}$$

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^{-1} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}, \tag{30}$$

where δ_t is a function of $(\Pi_{t+h})_{h=1}^{\infty}$ as defined in equation (20), and where $(D_t, \Pi_t)_{t=0}^{\infty}$ must satisfy the transversality condition in (22) given (27).

5 Main Results

Evaluating the dynamics around the steady state of our economy is challenging given the non-linear nature of the difference equations in (29)-(30). To present our main results, we consider the continuous-time limit of our model.[24] We derive this limit in Appendix A, where we introduce a generalized version of the model for an arbitrary time step $\mathrm{d}t$ and take the limit as $\mathrm{d}t \to 0$. Section 5.1 describes the system of equations defining an MPCE in the continuous-time limit. We characterize the steady state of the economy and the transition dynamics around the steady state in Section 5.2 and Section 5.3. In Section 5.4, we explore the quantitative implications of our model.

5.1 Continuous-Time Limit

Let $\lambda \equiv -\log(\theta)$ and $\rho \equiv -\log(\beta)$, and define $\pi_t \equiv \frac{\mathrm{d}}{\mathrm{d}t}\log(P_t)$ as the instantaneous rate of inflation at time t. Using (29)-(30) together with (20), Appendix A shows that the dynamics of price dispersion and inflation in the continuous-time limit of our model are given by

$$\dot{D}_t = \lambda \left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma\pi_t - \lambda)D_t, \tag{31}$$

$$\dot{\pi}_t = -\lambda\frac{\sigma(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma - 1)\pi_t], \tag{32}$$

where $\dot{\delta}_t = \delta_t^2 + [(\sigma - 1)\pi_t - (\rho + \lambda)]\delta_t$, and where $(D_t, \pi_t)_{t=0}^{\infty}$ must satisfy the continuous-time version of the transversality condition in (22) given (27):

$$\lim_{h\to\infty} e^{\left[-(\rho+\lambda)+\frac{\sigma}{h}\int_0^h \pi_{t+\ell}\mathrm{d}\ell\right]h}\left[\left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{1}{1-\sigma}} e^{-\int_0^h \pi_{t+\ell}\mathrm{d}\ell} - \frac{1+\tau}{D_{t+h}}\right] = 0. \tag{33}$$

5.2 Steady State

Our first main result establishes that there is a unique steady state in which price dispersion D_t and inflation π_t are constant and satisfy the system of equations (31)-(32) together with the transversality condition (33).[25] We define

[24]Taking the continuous-time limit is not necessary to perform comparative statics of the steady state, but it does facilitate the analysis of transition dynamics.

[25]The system (31)-(32) admits two solutions, but only one of them satisfies transversality.

$D_{ss}(\tau,\sigma)$ and $\pi_{ss}(\tau,\sigma)$ as the values of price dispersion and inflation in the steady state conditional on the labor wedge τ and the elasticity of substitution across varieties σ, and we study their comparative statics. Let us define

$$\overline{\tau}(\sigma) = \begin{cases} \infty & \text{if } \sigma \leq 2 \\ \frac{1}{\sigma^2 - 2\sigma} & \text{otherwise.} \end{cases}$$

We obtain the following result:[26]

Proposition 2. *There is a unique steady state* $(D_{ss}(\tau,\sigma), \pi_{ss}(\tau,\sigma))$*. Moreover,*

1. $D_{ss}(\tau,\sigma)$ *and* $\pi_{ss}(\tau,\sigma)$ *are both strictly increasing in the labor wedge* τ*.*
2. $D_{ss}(\tau,\sigma)$ *is strictly decreasing in the elasticity of substitution* σ *for* $\tau < \overline{\tau}(\sigma)$*, and* $\pi_{ss}(\tau,\sigma)$ *is strictly decreasing in* σ *for all* τ*.*

This proposition states that long-run price dispersion and inflation are higher the higher the labor wedge τ and the lower the elasticity of substitution across varieties σ (the latter holding for dispersion provided that $\tau < \overline{\tau}(\sigma)$). To understand these comparative statics, consider the incentives of the central bank starting from a given steady state. The central bank chooses a steady-state interest rate that sets the labor share to 1. Any consumption benefit from stimulating output beyond the steady-state level is exactly compensated by the cost of labor effort needed to do so. Now consider what happens following an unanticipated permanent increase in τ or decrease in σ. A central bank with commitment would be able to respond by preserving inflation stability, but this is not incentive compatible under lack of commitment.

For illustration, take the limiting case of Assumption 1 and suppose the economy begins in a steady state in which $\tau = -1/\sigma$. From (31)-(32) with $\dot{D}_t = \dot{\pi}_t = \dot{\delta}_t = 0$, the steady state admits zero price dispersion ($D = 1$) and zero inflation ($\pi = 0$), with a labor share $\mu = 1$. Suppose τ permanently increases. A central bank with commitment could preserve the levels of price dispersion and inflation by keeping nominal interest rates fixed forever. From

[26] We show in the proof of Proposition 2 that steady-state inflation satisfies $\pi_{ss}(\tau,\sigma) \in (0, \lambda/\sigma)$, which corresponds to $\Pi_{ss}(\tau,\sigma) \in (1, \theta^{-1/\sigma})$ in discrete time.

(24), the labor share would permanently decrease to satisfy

$$\mu = \frac{\sigma - 1}{\sigma(1+\tau)}$$

under the new higher level of τ.[27]

For a central bank without commitment, this policy is not incentive compatible. The reason is that it entails a reduction in the labor share, and the central bank has an incentive to undo the increase in monopoly distortions by stimulating output. If firms naively anticipated inflation stability, the central bank's best response would be to surprise markets by cutting interest rates.[28]

Flexible-price firms however are not naive. In equilibrium, they rationally forecast the monetary stimulus and the higher future labor demand and higher future real wages that ensue (relative to the commitment case). They also expect further inflation in the future. These expected future changes necessitate higher offsetting prices today. Over time, sequential price increases by flexible-price firms result in rising price dispersion. Eventually, the rise in dispersion reduces aggregate productivity sufficiently to offset the central bank's benefit from cutting interest rates, leading to a new steady state. Therefore, we obtain that both long-run price dispersion and long-run inflation are higher if the labor wedge τ is higher. Furthermore, the steady-state output and real wage (which are equal to each other) are lower under a higher labor wedge.

The intuition for a shock that permanently reduces the elasticity of substitution σ is similar. In this case, we establish the comparative static on long-run price dispersion under a positive upper bound on the labor wedge τ if $\sigma > 2$. The reason is that σ affects the law of motion of dispersion in (31); if $\tau > \overline{\tau}(\sigma)$, in principle dispersion could increase with σ. The comparative static on long-run inflation, instead, is unambiguous: a reduction in σ increases monopoly

[27] Analogous logic applies if σ decreases starting from $\tau = -1/\sigma$. More generally, the same reasoning applies with respect to the feasibility and the incentive incompatibility of inflation stabilization beginning from a positive-inflation steady state with $\tau > -1/\sigma$. The only difference in that case is that changes in σ require transition dynamics in dispersion and the labor share to support inflation stabilization.

[28] Formally, from the central bank's policy function (28), a reduction in Y_{t+1} (due to the reduction in the labor share) holding Π_{t+1} and D_t fixed requires a reduction in i_t.

distortions, and the central bank's response always leads to an increase in long-run inflation.

5.3 Transition Dynamics

Our second main result concerns the transition dynamics between steady states. We study an economy that transitions from an initial steady state to one with higher inflation following an unanticipated permanent shock. We show that inflation overshoots along the transition path to the new steady state.

Proposition 3. *Let $(D_{ss}(\tau,\sigma),\pi_{ss}(\tau,\sigma))$ be the steady state at time t_{ss}.*

1. *Consider the transition to steady state $(D_{ss}(\tau',\sigma),\pi_{ss}(\tau',\sigma))$ following an unanticipated shock that permanently increases the labor wedge to $\tau' > \tau$. There exists $t' \geq t_{ss}$ such $\pi_t > \pi_{ss}(\tau',\sigma)$ for all $t > t'$.*
2. *Consider the transition to steady state $(D_{ss}(\tau,\sigma'),\pi_{ss}(\tau,\sigma'))$ following an unanticipated shock that permanently decreases the elasticity of substitution to $\sigma' < \sigma$ given $\tau < \bar{\tau}(\sigma)$. There exists $t' \geq t_{ss}$ such $\pi_t > \pi_{ss}(\tau,\sigma')$ for all $t > t'$.*

Proposition 3 considers an unanticipated permanent shock that increases the labor wedge τ or reduces the elasticity of substitution across varieties σ. From Proposition 2, we know that long-run price dispersion and inflation increase in response to the shock. Proposition 3 tells us that inflation in the transition increases by more than in the long run; that is, transition dynamics involve inflation overshooting.

The proof of this result evaluates the three-dimensional phase diagram for price dispersion D_t, inflation π_t, and the auxiliary variable δ_t along a transition where D_t rises towards a higher steady-state level. To provide intuition, we next describe a special case of our model where we obtain a closed-form solution. Denote monopoly power by $\gamma \equiv \sigma(1+\tau)/(\sigma-1)$, where $\gamma > 1$ by Assumption 1. We consider a limit setting with $\sigma \to 1$ and τ adjusting so as to keep γ constant. In this limit, it can be shown (see Appendix C) that the auxiliary variable δ_t

must be constant at $\delta_t = \rho + \lambda$ for all t and the system (31)-(32) reduces to

$$\dot{D}_t = \lambda e^{-\frac{\pi_t}{\lambda}} + (\pi_t - \lambda) D_t, \tag{34}$$

$$\dot{\pi}_t = -\lambda (\rho + \lambda) \gamma e^{-\frac{\pi_t}{\lambda}} \frac{1}{D_t} + (\rho + \lambda - \pi_t) \lambda, \tag{35}$$

yielding simple expressions for the steady-state values. Moreover, given an initial steady state (D_{ss}, π_{ss}) and conditional on converging to a new steady state (D'_{ss}, π'_{ss}), we can solve for the dynamics for price dispersion and inflation:

$$\log(D_t) = \log(D'_{ss}) - \log\left(\frac{D'_{ss}}{D_{ss}}\right) e^{-\lambda t},$$

$$\pi_t = \pi'_{ss} + \lambda \log\left(\frac{D'_{ss}}{D_{ss}}\right) e^{-\lambda t}.$$

This solution shows the overshooting result of Proposition 3 explicitly. The transition following a shock that increases steady-state price dispersion to $D'_{ss} > D_{ss}$ has inflation converging to its new steady-state level π'_{ss} from above. In fact, in the limit setting we are describing here, inflation overshoots along the whole path: $\pi_t > \pi'_{ss}$ at all dates t in the transition. Inflation decays at the rate of λ, with the cumulative overshooting along the transition path given by[29]

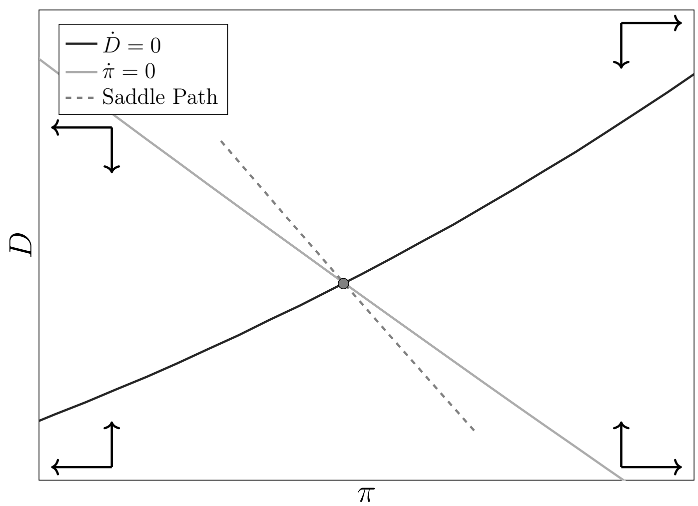
$$\int_0^\infty (\pi_t - \pi'_{ss}) dt = \log\left(\frac{D'_{ss}}{D_{ss}}\right). \tag{36}$$

Focusing on this limit setting, we can provide a simple graphical representation of the dynamics of our model. While the original system requires a three-dimensional phase diagram, taking $\sigma \to 1$ eliminates the dynamics of δ_t and reduces the dimensionality to $\mathbb{R}^2$. Figure 1 depicts the resulting phase diagram for system (34)-(35) in the neighborhood of some low steady-state inflation π_{ss}. The $\dot{\pi}_t = 0$ locus corresponds to the non-linear Phillips curve (35). This locus is downward sloping: higher inflation is sustained by lower price dispersion in a steady state, since lower dispersion increases output and

[29] Thus, a shock that increases price dispersion by Δ percent (i.e., with $\log(D'_{ss}/D_{ss}) = \Delta/100$) causes inflation to overshoot by $\lambda\Delta$ percent.

real wages, necessitating higher price increases by firms. Inflation increases (decreases) if dispersion is above (below) the locus. The $\dot{D}_t = 0$ locus corresponds to the dispersion dynamics equation (34). This locus is upward sloping: higher inflation is required to sustain higher price dispersion in a steady state, with the main forces being as discussed in our derivation of equation (19). Price dispersion increases (decreases) if inflation is above (below) the locus.

Figure 1: Phase Diagram for Inflation and Price Dispersion



Notes: This figure illustrates the phase diagram of the dynamic economy in the inflation-dispersion (π, D) plane in the limit setting with $\sigma \to 1$. The $\dot{D} = 0$ locus is upward sloping and the $\dot{\pi} = 0$ locus is downward sloping. The intersection point is the steady state. The flows of D and π are depicted by the arrows in the different regions of the plane. The directions of these flows indicate that any transition dynamics to the steady state should be along a saddle path with a negative slope.

The intersection of the $\dot{\pi}_t = 0$ and $\dot{D}_t = 0$ loci represents the steady state. As depicted in Figure 1, we show that the steady state of our economy admits a unique saddle path, and along this saddle path inflation and price dispersion evolve in opposite directions. The limit setting with $\sigma \to 1$ yields an explicit characterization of the saddle path:

$$\pi(D) = \pi_{ss} - \lambda \left(\log D - \log D_{ss}\right). \tag{37}$$

The negative slope of the saddle path reflects the central bank's incentives. For intuition, recall the scenario described in the previous section, where the economy is at a zero-inflation, zero-dispersion steady state with a labor share that has fallen strictly below 1. The central bank without commitment has an incentive to cut interest rates to stimulate output and reduce monopoly distortions. The economy must thus transition to a new steady state with strictly positive inflation and price dispersion. Along the transition path, the central bank sees a relatively larger benefit to stimulating output earlier on when price dispersion is low; anticipating this, flexible-price firms offset the ensuing higher wage costs with price increases that are also larger earlier on. The implication is rising price dispersion and declining inflation along the transition path.

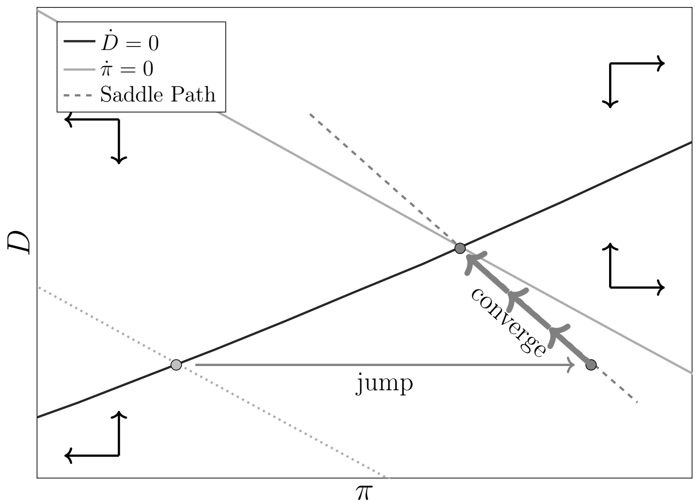
Figure 2 depicts the response to an unanticipated permanent increase in monopoly power γ, which would emerge from an increase in τ or decrease in σ. This shock does not affect the $\dot{D}_t = 0$ locus but shifts upward the $\dot{\pi}_t = 0$ locus: by (32), a higher level of price dispersion is needed to preserve a given level of inflation so as to offset the higher real wage costs. The new steady state following the shock is at the crossing point of the two loci, associated with higher inflation and higher price dispersion than in the initial crossing point.

Figure 2 shows that the transition to the new steady state involves inflation overshooting: inflation immediately jumps upward and then gradually declines towards its new steady-state level. This overshooting emerges because of the evolution of central bank incentives along the transition path. As discussed above, the central bank has a higher incentive to stimulate the economy earlier in the transition when price dispersion is low. Thus, inflation is high early in the transition and declines as dispersion rises along the path.

5.4 Quantitative Exploration

In this section, we study the quantitative implications of our analysis. We use a standard parameterization of the New Keynesian model and simulate a discrete-time economy in which every time period corresponds to a month.

Figure 2: Transition Dynamics



Notes: This figure illustrates the transition dynamics of dispersion and inflation following an unanticipated permanent increase in monopoly power γ in the limit setting with $\sigma \to 1$. The shock shifts the $\dot{\pi} = 0$ locus upwards while leaving the $\dot{D} = 0$ locus unchanged. Inflation jumps on impact to move the economy to its new saddle path, after which D increases and π declines towards their new steady-state levels. The transition involves inflation overshooting.

We take a discount factor $\beta = (1.02)^{-1/12}$ to target a steady-state annual real interest rate of 2 percent. The probability that a firm has sticky prices is set at $\theta = 0.86$ to target an average duration of price stickiness of 7 months (e.g., Nakamura and Steinsson, 2008). The elasticity of substitution across varieties is set at $\sigma = 7$, in line with previous research on the cost of inflation (e.g., Coibion, Gorodnichenko, and Wieland, 2012). The inverse elasticity of labor supply is set at $\psi = 2.5$, which is in the range of estimates in the literature (e.g., Chetty, Guren, Manoli, and Weber, 2011).[30] Finally, for the labor wedge, we specify $\tau = -0.1427$ to target a steady-state annual inflation rate of 2 percent under central bank lack of commitment.[31] Table 1 summarizes our choice of parameters.

[30] Observe that this choice has no bearing on our findings under lack of commitment, since ψ does not enter the dynamic equations characterizing our economy. This value only affects the findings under commitment to inflation targeting and the estimates of welfare that we discuss at the end of this section.

[31] This value is uniquely pinned down given the comparative statics in Proposition 2.

Table 1: Parameters

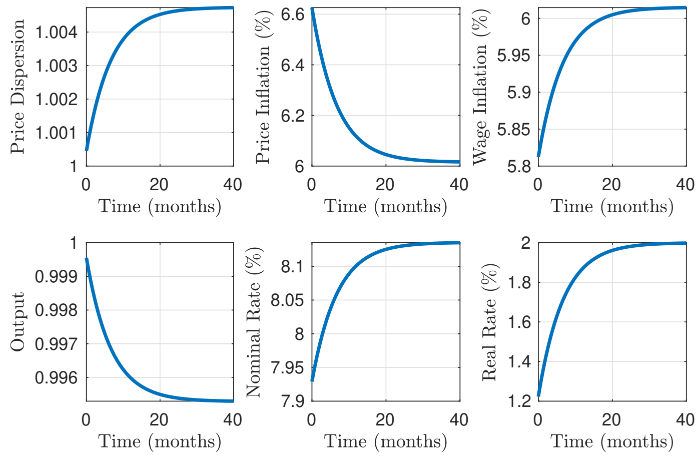
Parameter	**Value**	**Target**
Discount factor, β	$(1.02)^{-\frac{1}{12}}$	2% annual real interest rate
Fraction of sticky-price firms, θ	0.86	Nakamura and Steinsson (2008)
Elasticity of substitution, σ	7	Coibion, Gorodnichenko, and Wieland (2012)
Inverse Frisch elasticity, ψ	2.5	Chetty, Guren, Manoli, and Weber (2011)
Labor wedge, τ	-0.1427	2% annual inflation without commitment

Starting from the steady state of the economy given the parameter values in Table 1, Figure 3 considers an unanticipated permanent increase in the labor wedge τ that takes the economy to a new steady state with 6-percent annual inflation. The figure displays the transition paths of price dispersion, real output, the price inflation rate, the nominal wage growth rate, the nominal interest rate, and the real interest rate, where the monthly values for the latter four variables are represented in annualized form.

In line with our analytical results, Figure 3 shows that inflation overshoots following the shock by immediately jumping up from its initial 2-percent level (not shown in the figure given the scale) and then gradually declining towards its new higher steady-state level. The nominal interest rate jumps up and continues to increase throughout the transition, while the real interest rate jumps down (since the central bank initially stimulates the economy to weather the shock) and then gradually returns to its original level. Along the transition path, output gradually falls as price dispersion and labor misallocation increase. Nominal wage inflation jumps up initially in tandem with price inflation, and it then gradually converges to a permanently higher level. Note that wage inflation is below price inflation; these dynamics underpin the permanent long-run decline in the real wage.

We find that a small change in the labor wedge has a sizable impact on long-run inflation. For the annualized steady-state inflation rate to increase from 2 to 6 percent, the labor wedge τ must increase from -0.1427 to only -0.1424. Moreover, as shown in Figure 3, the shock causes inflation to overshoot to 6.6

Figure 3: Response to Unanticipated Increase in Labor Wedge



Notes: This figure shows the transition dynamics following an unanticipated permanent increase in the labor wedge τ that takes the economy from an initial steady state with 2-percent inflation (not shown in the figure given the scale) to a new steady state with 6-percent inflation. The inflation, interest, and wage growth rates are annualized (annual inflation $= e^{12\pi_t} - 1$).

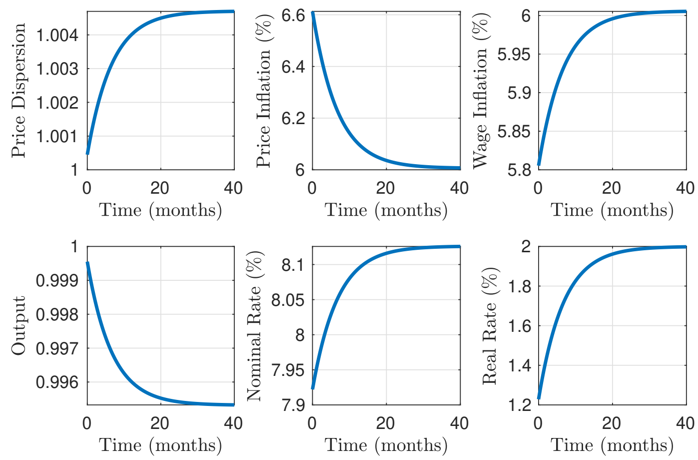
percent on impact, and this inflation overshooting is persistent. Following the jump, it takes 6 months for the inflation rate to decline within 25 basis points of its new steady-state level of 6 percent, and a total of 16.5 months for the inflation rate to decline within 5 basis points of that steady-state level.

The dynamics in Figure 3 are markedly different from those that would arise under inflation targeting, namely if the central bank was committed to maintaining a 2-percent inflation level in every period. Under inflation targeting, following an increase in the labor wedge, the central bank would keep real and nominal interest rates fixed so as to preserve the level of inflation. Output would immediately decline following the shock and would remain at a lower level. Price dispersion would not change in response to the shock.

Figure 4 presents an analogous exercise to that in Figure 3 by considering an unanticipated permanent decrease in the elasticity of substitution σ that takes the economy from its initial 2-percent-inflation steady state to a new steady state with 6-percent inflation.[32] The transition dynamics are similar as

[32]The elasticity of substitution σ decreases from 7 to 6.9834 in this exercise.

Figure 4: Response to Unanticipated Decrease in Elasticity of Substitution



Notes: This figure shows the transition dynamics following an unanticipated permanent decrease in the elasticity of substitution σ that takes the economy from an initial steady state with 2-percent inflation (not shown in the figure given the scale) to a new steady state with 6-percent inflation. The inflation, interest, and wage growth rates are annualized (annual inflation $= e^{12\pi_t} - 1$).

in the case of a positive labor wedge shock and are in line with our analytical results. As for the contrast with the dynamics that would arise under inflation targeting, things are different when the shock is to σ rather than τ. The reason is that the change in the elasticity of substitution σ directly affects the dynamic relationship between price dispersion and inflation. If the central bank was committed to maintaining a 2-percent inflation level in every period, then following a decrease in σ, steady-state price dispersion would decline,[33] and the real and nominal interest rates would evolve so as to facilitate the transition of the economy to the lower dispersion level.

The exercises above provide a framework for evaluating the welfare benefits of inflation targeting relative to our central bank's policy under lack of commitment. Given an unanticipated permanent shock, the benefit of inflation targeting over the no-commitment policy is that it reduces the misallocation cost

[33] This is because greater differentiation across varieties means that relative price differences are a less important source of misallocation.

Table 2: Inflation Targeting versus No Commitment

Scenario	Welfare under Targeting	Welfare under No Commitment	Welfare Difference
τ shock	0.982	0.950	0.032
σ shock	0.982	0.950	0.032

Notes: The table reports welfare expressed in consumption-equivalent terms relative to an identical economy with flexible prices. That is, we compute, right after each shock, how much a household would require in consumption to be indifferent between living in an economy with sticky prices and inflation targeting/no commitment and living in an economy with flexible prices where consumption and its implied labor supply are forever constant at the offered level.

of long-run price dispersion in the economy. The benefit of the no-commitment policy is that it reduces the short-run and long-run costs of rising monopoly distortions. We compare welfare under each regime in Table 2, where welfare is expressed in consumption-equivalent terms relative to an identical economy with flexible prices. The table considers the two scenarios studied in Figure 3 and Figure 4, namely an unanticipated permanent increase in the labor wedge τ and decrease in the elasticity of substitution σ, each computed so that the economy transitions from its initial 2-percent-inflation steady state to a new steady state with 6-percent inflation.

We find that in both scenarios, welfare under inflation targeting is strictly higher than under lack of commitment. Moreover, the welfare gains from inflation targeting are substantial, at about 3 percent in consumption-equivalent terms.[34] In other words, the long-run price dispersion costs under lack of commitment far outweigh the benefits from reducing monopoly distortions, and the high discount factor β implies that these costs enter prominently in the welfare calculation. The analysis suggests that there can be significant benefits to institutions that enhance commitment to inflation targeting.

[34]Theoretically, the welfare effects are different in the two scenarios because an increase in τ only affects the labor wedge, whereas a decrease in σ also affects the aggregation of the model. However, the aggregation component is negligible in our simulations for a small shock to σ. As a result, Table 2 reports numerically identical effects in the two scenarios.

The large impact of small shocks to τ and σ, both on inflation dynamics and on welfare relative to inflation targeting, is a robust feature of our model. It emerges because the steady-state labor share is relatively insensitive to inflation; much of the positive effect of inflation on the labor share via sticky-price firms is offset by the negative effect via forward-looking flexible-price firms.[35] In fact, note that standard calibrations of the New Keynesian model take high values of β and low values of θ. This means that in response to monetary stimulus, there is a large number $1-\theta$ of flexible-price firms which raise prices significantly to protect against potentially overhiring in the future, and this puts downward pressure on the labor share.[36] As a result, a central bank without commitment—which seeks to keep the labor share from declining—ends up increasing inflation substantially in response to a small increase in the labor wedge or a small reduction in the elasticity of substitution.

To see this formally, consider the *long-run* Phillips curve, derived by combining steady-state equations (23) and (24) to express the relationship between the long-run labor share μ and long-run inflation Π:[37]

$$\mu = \frac{\sigma-1}{\sigma(1+\tau)}\left[1+(1-\beta)\frac{\theta\Pi^{\sigma-1}(\Pi-1)}{(1-\theta\Pi^{\sigma})(1-\beta\theta\Pi^{\sigma-1})}\right]. \tag{38}$$

If the discount factor β is close to 1, then the term in brackets on the right-hand side of (38) is relatively insensitive to Π. In this case, the labor share μ does not respond significantly to inflation and the long-run Phillips curve is almost vertical. As a consequence, small changes in the labor wedge τ or the elasticity of substitution σ require large changes in inflation Π to keep the labor share μ constant in (38). This explains the large quantitative magnitudes in our model.

There are several implications that follow from this discussion. First, any changes to parameters or to the underlying price-setting mechanism that

[35] See the discussion following Lemma 2 in Section 3.2.

[36] A low value of θ also implies that flexible-price firms place a low probability on the likelihood of not being able to adjust prices in the future. While this implies a low anticipatory channel for each individual flexible-price firm, this effect is offset by the fact that a low θ implies a large share of flexible-price firms.

[37] We express these in discrete time for expositional symmetry with the discussion in Section 3.2, but the same point can be made using the continuous-time representation.

result in a flatter long-run Phillips curve would imply smaller quantitative magnitudes in our model. Second, such changes would also imply a lower value of commitment to inflation targeting, since the central bank's lack of commitment would then have a smaller effect on equilibrium inflation and price dispersion. Finally, changes that yield a flatter long-run Phillips curve would also imply meaningful economic benefits from long-run inflation, indicating that inflation targeting at too low an inflation rate would be costly for society.

6 Concluding Remarks

In this paper, we introduced central bank lack of commitment into a standard non-linear New Keynesian model with monopolistically competitive firms and sticky prices. We characterized long-run inflation and studied transition dynamics as the economy responds to an unanticipated permanent shock that increases the labor wedge or decreases the elasticity of substitution across varieties. While a central bank with commitment would be able to keep inflation unchanged, inflation stabilization is not incentive compatible for a central bank without commitment. The private sector anticipates central bank accommodation following the shock, and inflation overshoots before declining to a permanently higher level. These effects are quantitatively large, as is the welfare loss from lack of commitment relative to inflation targeting.

Our framework is useful in interpreting the inflationary spike that has befallen advanced economies in the aftermath of the COVID-19 pandemic. Many questions have emerged regarding the causes of this inflation and whether central banks will ultimately be successful in bringing inflation back down to pre-pandemic levels. In Afrouzi, Halac, Rogoff, and Yared (2024), we use the framework of this paper to shed light on the factors that contributed to the decline in global inflation over the four decades before the pandemic. We argue that globalization, the proliferation of the Washington consensus, and deunionization led to a reduction in firm monopoly power and labor market power that lowered pressures on central banks to inflate. These trends, however, appear to be reversing themselves in the post-pandemic period. We argue that deglobalization, rising fiscal pressures, and rising long-term real interest rates will likely increase central banks' incentives to inflate and stimulate the economy. Absent a strengthened commitment to inflation stability, our framework predicts higher average inflation in the coming decade compared to the past, with occasional bursts of elevated inflation due to overshooting.

Our analysis leaves a number of avenues for future research. First, while we have examined the canonical New Keynesian model with Calvo pricing, our approach can be applied to other models of price setting, such as menu-cost models. The optimal policy of the central bank without commitment—which sets the labor share to 1—is invariant to the details of the underlying price-setting model, and future research can use our approach to explore how inflation dynamics might change under different models. Second, by focusing on the stable steady state, we have ignored broader issues involving equilibrium implementation and off-equilibrium inflation stability (see, e.g., Woodford, 2003; Atkeson, Chari, and Kehoe, 2010; Cochrane, 2011; Galí, 2015; Neumeyer and Nicolini, 2022). A natural question concerns the extent to which lack of commitment on and off the equilibrium path increases or decreases the scope for off-equilibrium inflation stability in our framework. Finally, our model abstracts from monetary and fiscal interactions by assuming lump sum taxes and Ricardian equivalence. It would be interesting to relax the assumption of Ricardian equivalence and study how central bank lack of commitment interacts with fiscal lack of commitment. Since our analytical framework does not assume a long-run level of debt (as it is not linearized), it can facilitate such an analysis.

Appendix

A Continuous-Time Limit

In this appendix, we solve the discrete-time model for an arbitrary time step of length $\mathrm{d}t$ and derive the continuous-time limit as $\mathrm{d}t \to 0$. For completeness, we first reiterate the derivations of the discrete-time model for a given $\mathrm{d}t$, where $\mathrm{d}t = 1$ corresponds to the derivations in the main text.

Time now runs at increments of $\mathrm{d}t$, so that $t \in T_{\mathrm{d}t} \equiv \{0, \mathrm{d}t, 2\mathrm{d}t, \ldots\}$. Letting $\rho \equiv -\log(\beta)$, the household's problem for a given $\mathrm{d}t$ is given by

$$\max_{C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j\in[0,1]}} \sum_{t \in T_{\mathrm{d}t}} e^{-\rho t} \left(\log(C_t) - \frac{L_t^{1+\psi}}{1+\psi} \right) \mathrm{d}t$$

subject to

$$\int_0^1 P_{j,t} C_{j,t} \mathrm{d}j \mathrm{d}t + B_t \leq W_t L_t \mathrm{d}t + (1 + i_{t-\mathrm{d}t} \mathrm{d}t) B_{t-\mathrm{d}t} + \int_0^1 s_{j,t} X_{j,t} \mathrm{d}t \mathrm{d}j + \int_0^1 (s_{j,t-\mathrm{d}t} - s_{j,t}) P^S_{j,t} \mathrm{d}j - T_t \mathrm{d}t,$$

$$C_t = \left(\int_0^1 C_{j,t}^{1-\sigma^{-1}} \mathrm{d}j \right)^{\frac{1}{1-\sigma^{-1}}}.$$

Note that this expression of the problem redefines $C_{j,t}$, C_t, L_t, $X_{j,t}$ and T_t as *rates* of consumption, labor supply, profits, and lump-sum taxes per $\mathrm{d}t$.

The implied demand for varieties $j \in [0, 1]$, the definition of the aggregate price P_t, the price dispersion measure D_t, and the intratemporal labor supply condition are all identical to those in the main text because they follow from static decisions that are not affected by the time step $\mathrm{d}t$. To reiterate these, we have

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t} \right)^{1-\sigma}, \forall j, \quad P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} \mathrm{d}j \right)^{\frac{1}{1-\sigma}}, \quad D_t = \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} \mathrm{d}j, \quad \frac{W_t}{P_t} = C_t L_t^{\psi}.$$

Moreover, under the assumptions in the main text—in particular, the fact that firms produce with $Y_{j,t} = L_{j,t}$ and always produce enough to meet their demand—we can still use the labor market clearing conditions to derive the aggregate production function of the economy as

$$L_t = \int_0^1 L_{j,t}\mathrm{d}j = \int_0^1 C_{j,t}\mathrm{d}j = C_t \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma} \mathrm{d}j = C_t D_t \implies C_t = \frac{L_t}{D_t},$$

where D_t is defined as the price dispersion measure similar to the main text.

The Euler equations for nominal bonds and stocks, however, are affected by the time step and are given by

$$\begin{aligned} \frac{1}{P_t C_t} &= e^{-\rho \mathrm{d}t}(1 + i_t \mathrm{d}t)\frac{1}{P_{t+\mathrm{d}t}C_{t+\mathrm{d}t}}, \\ P^S_{j,t} &= X_{j,t}\mathrm{d}t + \frac{1}{1 + i_t \mathrm{d}t} P^S_{j,t+\mathrm{d}t}, \forall j. \end{aligned}$$

Rearranging these, we obtain the following expressions:[38]

$$\frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} = i_t - \rho, \qquad \dot{P}^S_{j,t} = i_t P^S_{j,t} - X_{j,t}, \quad \forall j \in [0,1]$$

where for any variable X_t, we define $\dot{X}_t$ as their rate of change over time, i.e., $\dot{X}_t \equiv \mathrm{d}X_t/\mathrm{d}t$. Integrating the Euler equation for stocks forward and assuming no bubbles gives us the household's valuation of firms at time t:

$$P^S_{j,t} = \int_0^\infty e^{-\int_0^h i_{t+s}\mathrm{d}s} X_{j,t+h}\mathrm{d}h = \int_0^\infty e^{-\rho h}\frac{P_t C_t}{P_{t+h}C_{t+h}} X_{j,t+h}\mathrm{d}h.$$

We will use this valuation to rewrite the optimization problem of a firm that gets the opportunity to reset its price. Before we do so, we have to adjust the frequency of price changes such that the probability of changing prices is independent of the choice of $\mathrm{d}t$. To this end, let $\theta^{\mathrm{d}t}$ be the probability of not getting the opportunity to adjust the price at an interval of length $\mathrm{d}t$. This defines a consistent distribution of price adjustment frequency for different values of $\mathrm{d}t$ such that for any interval length T, the probability of not adjusting the price is θ^T, independent of $\mathrm{d}t$. With $T = 1$, this corresponds to the model in the main text where $\mathrm{d}t = 1$. With $\mathrm{d}t \to 0$, it corresponds to

[38] These expressions follow from dividing the equations above by $\mathrm{d}t$ and taking the limit as $\mathrm{d}t \to 0$.

a Poisson process where the arrival rate of price adjustment opportunities is $\lambda \equiv -\log(\theta)$. We obtain a well-defined limit: under the Poisson arrival rate of λ, the implied distribution of time between price changes is exponential with scale λ. Accordingly, the probability of not adjusting the price in a period of length T is $e^{-\lambda T} = e^{\log(\theta)T} = \theta^T$.

Now, for a given $\mathrm{d}t$, a flexible-price firm's problem for choosing its reset price is given by maximizing the net present value of its profits in the history where it is stuck with the price it chooses at date t:

$$\max_{P_t^*} \sum_{h \in T_{\mathrm{d}t}} e^{-(\rho+\lambda)h} \frac{P_t C_t}{P_{t+h} C_{t+h}} [P_t^* - (1+\tau) W_{t+h}] C_t \left(\frac{P_t^*}{P_{t+h}} \right)^{-\sigma} \mathrm{d}t.$$

The first-order condition for P_t^* is

$$\sum_{h \in T_{\mathrm{d}t}} e^{-(\rho+\lambda)h} \frac{P_t C_t}{P_{t+h} C_{t+h}} C_{t+h} P_{t+h}^{\sigma} \left[P_t^* - \frac{\sigma(1+\tau)}{\sigma - 1} W_{t+h} \right] \mathrm{d}t = 0,$$

which, following the main text, can be simplified and rewritten as

$$\frac{P_t^*}{P_t} = \frac{\sigma(1+\tau)}{\sigma-1} \frac{\sum_{h \in T_{\mathrm{d}t}} e^{-(\rho+\lambda)h} \left(\frac{P_{t+h}}{P_t} \right)^{\sigma} \frac{W_{t+h}}{P_{t+h}} \mathrm{d}t}{\sum_{h \in T_{\mathrm{d}t}} e^{-(\rho+\lambda)h} \left(\frac{P_{t+h}}{P_t} \right)^{\sigma-1} \mathrm{d}t}. \tag{A.1}$$

We can again define the auxiliary variable δ_t as the inverse of the denominator in (A.1), which can be written recursively as

$$\delta_t^{-1} \equiv \sum_{h \in T_{\mathrm{d}t}} e^{-(\rho+\lambda)h} \left(\frac{P_{t+h}}{P_t} \right)^{\sigma-1} \mathrm{d}t = \mathrm{d}t + e^{-(\rho+\lambda)\mathrm{d}t} \left(\frac{P_{t+\mathrm{d}t}}{P_t} \right)^{\sigma-1} \delta_{t+\mathrm{d}t}^{-1}. \tag{A.2}$$

Similarly, we can write (A.1) recursively as

$$\begin{aligned} \frac{P_t^*}{P_t} &= \frac{\sigma(1+\tau)}{\sigma-1} \frac{W_t}{P_t} \delta_t \mathrm{d}t + e^{-(\rho+\lambda)\mathrm{d}t} \left(\frac{P_{t+\mathrm{d}t}}{P_t} \right)^{\sigma} \frac{\delta_t}{\delta_{t+\mathrm{d}t}} \frac{P_{t+\mathrm{d}t}^*}{P_{t+\mathrm{d}t}} \\ &= \frac{\sigma(1+\tau)}{\sigma-1} \frac{W_t}{P_t} \delta_t \mathrm{d}t + (1 - \delta_t \mathrm{d}t) \frac{P_{t+\mathrm{d}t}}{P_t} \frac{P_{t+\mathrm{d}t}^*}{P_{t+\mathrm{d}t}}, \end{aligned} \tag{A.3}$$

where the second line follows from substituting (A.2) in (A.3).

Next, we can derive the aggregate price as

$$P_t^{1-\sigma} = \int_0^1 P_{i,t}^{1-\sigma} \mathrm{d}j = (1 - e^{-\lambda \mathrm{d}t})(P_t^*)^{1-\sigma} + e^{-\lambda \mathrm{d}t} P_{t-\mathrm{d}t}^{1-\sigma},$$

where we have used the property that the set of firms with sticky prices are a random sample of the population at each instant. This equation implies the following relationship between relative reset price and gross inflation rate:

$$1 = (1 - e^{-\lambda \mathrm{d}t}) \left(\frac{P_t^*}{P_t} \right)^{1-\sigma} + e^{-\lambda \mathrm{d}t} \left(\frac{P_t}{P_{t-\mathrm{d}t}} \right)^{\sigma-1}.$$

Defining $\pi_t \equiv \frac{1}{\mathrm{d}t} \log(P_t / P_{t-\mathrm{d}t})$ as the rate of inflation at time t, we can rewrite this equation as

$$\frac{P_t^*}{P_t} = \left[\frac{1 - e^{[(\sigma-1)\pi_t - \lambda]\mathrm{d}t}}{1 - e^{-\lambda \mathrm{d}t}} \right]^{\frac{1}{1-\sigma}},$$

which is the equivalent of equation (18) in the main text once we set $\mathrm{d}t = 1$ and plug $\theta = e^{-\lambda}$. Moreover, using this equation, combined with the intratemporal labor supply condition and the aggregate production function $C_t = Y_t = L_t / D_t$, equations (A.2) and (A.3) become

$$\delta_t^{-1} = \mathrm{d}t + e^{[(\sigma-1)\pi_{t+\mathrm{d}t} - (\rho+\lambda)]\mathrm{d}t} \delta_{t+\mathrm{d}t}^{-1}, \tag{A.4}$$

$$\left[\frac{1 - e^{[(\sigma-1)\pi_t - \lambda]\mathrm{d}t}}{1 - e^{-\lambda \mathrm{d}t}} \right]^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma - 1} Y_t^{1+\psi} D_t^{\psi} \delta_t \mathrm{d}t + (1 - \delta_t \mathrm{d}t) e^{\pi_{t+\mathrm{d}t} \mathrm{d}t} \left[\frac{1 - e^{[(\sigma-1)\pi_{t+\mathrm{d}t} - \lambda]\mathrm{d}t}}{1 - e^{-\lambda \mathrm{d}t}} \right]^{\frac{1}{1-\sigma}}, \tag{A.5}$$

which are the equivalents of equations (20) and (21) in the main text, respectively.

We next write the equation for the price dispersion dynamics. By random selection of price-setters at any given t, we can write this equation as

$$D_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma} \mathrm{d}j = (1 - e^{-\lambda \mathrm{d}t}) \left(\frac{P_t^*}{P_t}\right)^{-\sigma} + e^{-\lambda \mathrm{d}t} \left(\frac{P_t}{P_{t-\mathrm{d}t}}\right)^{\sigma} \int_0^1 \left(\frac{P_{j,t-\mathrm{d}t}}{P_{t-\mathrm{d}t}}\right)^{-\sigma} \mathrm{d}j$$
$$= (1 - e^{-\lambda \mathrm{d}t}) \left[\frac{1 - e^{[(\sigma-1)\pi_t - \lambda]\mathrm{d}t}}{1 - e^{-\lambda \mathrm{d}t}}\right]^{\frac{\sigma}{\sigma-1}} + e^{\sigma \pi_t \mathrm{d}t - \lambda \mathrm{d}t} D_{t-\mathrm{d}t}. \tag{A.6}$$

Finally, we can write the central bank's problem with a general time step as follows:

$$V(\Omega_t) = \max_{D_t, L_t} \left\{ \log(D_t) - \frac{L_t^{1+\psi}}{1+\psi} + e^{-\rho \mathrm{d}t} V(\Omega_{t+\mathrm{d}t}) \right\} \quad \text{subject to} \quad Y_t = \frac{L_t}{D_t},$$

which gives the same optimal policy as in the main text, $Y_t = 1/D_t$. This policy implies that the real wage from the intratemporal labor supply condition is given by

$$\frac{W_t}{P_t} = Y_t L_t^{\psi} = Y_t^{1+\psi} D_t^{\psi} = \frac{1}{D_t}.$$

Plugging this optimal policy into equation (A.5) and taking the limit as $\mathrm{d}t \to 0$ in equations (A.4) to (A.6), we obtain the continuous-time analogs of the equations that characterize D_t, π_t and δ_t, as presented in the main text:

$$\dot{D}_t = \lambda \left(1 - \frac{\sigma - 1}{\lambda} \pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma \pi_t - \lambda) D_t,$$
$$\dot{\pi}_t = -\lambda \frac{\sigma(1+\tau)}{\sigma - 1} \left(1 - \frac{\sigma - 1}{\lambda} \pi_t\right)^{\frac{\sigma}{\sigma-1}} \frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma - 1)\pi_t],$$
$$\dot{\delta}_t = \delta_t^2 + [(\sigma - 1)\pi_t - (\rho + \lambda)]\delta_t.$$

B Proofs

B.1 Proof of Lemma 1

Take an initial price distribution $(P_{j,-1})_{j\in[0,1]}$ and a sequence of policies $(i_t)_{t=0}^{\infty}$. The arguments in the text show that if a sequence of allocations and prices $(L_t, Y_t, D_t, \delta_t, \Pi_t)_{t=0}^{\infty}$ is supported by a competitive equilibrium, then it satisfies conditions (13), (14), (19), (20), (21), and (22). This proves the necessity claim.

To prove the sufficiency claim, suppose that a sequence $(L_t, Y_t, D_t, \delta_t, \Pi_t)_{t=0}^{\infty}$ satisfies conditions (13), (14), (19), (20), (21), and (22) given $(P_{j,-1})_{j\in[0,1]}$ and $(i_t)_{t=0}^{\infty}$. The set $(P_{j,-1})_{j\in[0,1]}$ defines P_{-1}, and we can define $P_t = \Pi_{t-1}P_{t-1}$ recursively. Let $P_{j,t} = P_{j,t-1}$ if firm j cannot change prices at t, and $P_{j,t} = P_t^*$ if the firm can change prices at t, where P_t^* is given by (18). Define W_t according to (16) and let $B_t = 0$ at all dates with T_t chosen to satisfy (11). Letting $C_t = Y_t$, define $C_{j,t}$ according to (2), and let $Y_{j,t} = L_{j,t} = C_{j,t}$. Additionally, let

$$X_{j,t} = [P_{j,t} - (1+\tau)W_t]C_t \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma},$$

define $P_{j,t}^S$ according to (6), and let $s_{j,t} = 1$ so that the representative household holds a share of every firm $j \in [0,1]$. The household's problem (1) is concave and yields a unique solution. It can be verified that the values of $(C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j\in[0,1]})_{t=0}^{\infty}$ satisfy all optimality conditions of the household's problem, with the transversality condition being verified below. The firm's problem (9) is concave and yields a unique solution. It can be verified that the values of $(P_t^*, Y_{j,t}, L_{j,t})_{t=0}^{\infty}$ satisfy all optimality conditions of the firm's problem. Therefore, we conclude that the sequence $(L_t, Y_t, D_t, \delta_t, \Pi_t)_{t=0}^{\infty}$ supports a competitive equilibrium.

We next verify the transversality condition. Consider the date-t price of an Arrow-Debreu security that pays a coupon equal to firm j's profits at date $t+h$ for $h > 0$. There are three cases to consider. First, suppose the firm's price has always been sticky. Then the probability of arriving at such a history at $t+h$ from the perspective of date t is θ^h, and the price that the firm is charging at $t+h$ is $P_{j,-1}$. Appealing to the intertemporal condition, we can write the limiting price of the Arrow-Debreu security at date t as $h \to \infty$ as

$$\lim_{h\to\infty} \beta^h \theta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_{j,-1} - (1+\tau) W_{t+h}\right] C_{t+h} \left(\frac{P_{j,-1}}{P_{t+h}}\right)^{-\sigma} = 0, \tag{B.1}$$

where transversality requires that this price go to zero.

Second, suppose the firm's price has been sticky since date ℓ for $0 \le \ell \le t$. Then the probability of arriving at such a history at $t+h$ from the perspective of date t is θ^h, and the price that the firm is charging at $t+h$ is P_ℓ^*. The transversality condition in this case is

$$\lim_{h\to\infty} \beta^h \theta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_\ell^* - (1+\tau) W_{t+h}\right] C_{t+h} \left(\frac{P_\ell^*}{P_{t+h}}\right)^{-\sigma} = 0. \tag{B.2}$$

Finally, suppose the firm's price has been sticky since date $\ell > t$. Then the probability of arriving at such a history at $t+h$ from the perspective of date t is $(1-\theta)\,\theta^{t+h-\ell}$, and the price that the firm is charging at $t+h$ is P_ℓ^*. The transversality condition in this case is

$$\lim_{h\to\infty} \beta^h (1-\theta)\, \theta^{t+h-\ell} \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_\ell^* - (1+\tau) W_{t+h}\right] C_{t+h} \left(\frac{P_l^*}{P_{t+h}}\right)^{-\sigma} = 0. \tag{B.3}$$

To verify that (B.2) and (B.3) are satisfied, note that we can multiply (B.2) by $\beta^{-\ell}\theta^{-\ell} P_\ell C_\ell / P_t C_t$ without changing its limit as $h \to \infty$, which means that satisfaction of (B.2) is equivalent to

$$\lim_{h\to\infty} \beta^{h-\ell} \theta^{h-\ell} \frac{P_\ell C_\ell}{P_{t+h} C_{t+h}} \left[P_\ell^* - (1+\tau) W_{t+h}\right] C_{t+h} \left(\frac{P_\ell^*}{P_{t+h}}\right)^{-\sigma} = 0. \tag{B.4}$$

Similarly, we can multiply (B.3) by $(1-\theta)^{-1}\theta^{-t} P_\ell C_\ell / P_t C_t$ without changing its limit as $h \to \infty$, which means that satisfaction of (B.3) is also equivalent to (B.4). Moreover, observe that given (15), (16), and (18), and noting that $P_t C_t \left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{-\sigma} > 0$, it follows that satisfaction of (22) implies satisfaction of (B.4). Hence, (B.2) and (B.3) are both satisfied.

We are left to verify that (B.1) is also satisfied. We can multiply (B.1) by $P_{j,-1}^{\sigma}/P_t C_t$ without changing its limit as $h \to \infty$, which means that satisfaction of (B.1) is equivalent to

$$\lim_{h\to\infty} \beta^h \theta^h P_h^{\sigma} \left[\left(\frac{P_{j,-1}}{P_{-1}} \right) \frac{P_{-1}}{P_h} - (1+\tau) \frac{W_h}{P_h} \right] = 0.$$

Under the constructed equilibrium, this limit can be rewritten as

$$\lim_{h\to\infty} \left[\beta\theta \left(\prod_{\ell=0}^{h} \Pi_\ell \right)^{\frac{\sigma}{h}} \right]^h \left[\left(\frac{P_{j,-1}}{P_{-1}} \right) \frac{1}{\prod_{\ell=0}^{h} \Pi_\ell} - (1+\tau) Y_h^{\psi} D_h^{1+\psi} \right] = 0. \quad \text{(B.5)}$$

There are two possible cases. Suppose first that $\lim_{h\to\infty} \left[\beta\theta \left(\prod_{\ell=0}^{h} \Pi_\ell \right)^{\frac{\sigma}{h}} \right]^h = 0$. Then note that by (B.4) for $\ell = 0$, the second bracket stays finite as $h \to \infty$. Hence, in this case, (B.5) and thus (B.1) are satisfied.

Suppose next that $\lim_{h\to\infty} \left[\beta\theta \left(\prod_{\ell=0}^{h} \Pi_\ell \right)^{\frac{\sigma}{h}} \right]^h \neq 0$. Then satisfaction of (B.4) for $\ell = 0$ implies

$$\lim_{h\to\infty} \left[\frac{P_0^*}{P_0} \frac{1}{\prod_{\ell=0}^{h} \Pi_\ell} - (1+\tau) Y_h^{\psi} D_h^{1+\psi} \right] = 0.$$

It follows that if (B.5) is not satisfied, then we must have

$$\lim_{h\to\infty} \left[\frac{P_0^*}{P_0} \frac{1}{\prod_{\ell=0}^{h} \Pi_\ell} - (1+\tau) Y_h^{\psi} D_h^{1+\psi} - \left(\frac{P_{j,-1}}{P_{-1}} \right) \frac{1}{\prod_{\ell=0}^{h} \Pi_\ell} - (1+\tau) Y_h^{\psi} D_h^{1+\psi} \right] \neq 0,$$

or, equivalently,

$$\lim_{h\to\infty} \left\{ \left[\frac{P_0^*}{P_0} - \left(\frac{P_{j,-1}}{P_{-1}} \right) \frac{P_0}{P_1} \right] \frac{1}{\prod_{\ell=0}^{h} \Pi_\ell} \right\} \neq 0.$$

But this means that $\frac{1}{\prod_{\ell=0}^{h} \Pi_\ell}$ does not approach zero as $h \to \infty$, which contradicts the assumption that $\lim_{h\to\infty}\left[\beta\theta\left(\prod_{\ell=0}^{h}\Pi_\ell\right)^{\frac{\sigma}{h}}\right]^h \neq 0$. Hence, (B.5) and thus (B.1) are satisfied.

B.2 Proof of Lemma 2

Consider first price dispersion D. Equation (23) defines D as a function of Π in the steady state. Differentiating this equation yields

$$\begin{aligned}\frac{\partial}{\partial \Pi}D &= \theta\sigma D\Pi^{\sigma-2}\left(-\frac{1}{1-\theta\Pi^{\sigma-1}}+\frac{\Pi}{1-\theta\Pi^{\sigma}}\right)\\ &= \theta\sigma D\Pi^{\sigma-2}\frac{\Pi-1}{(1-\theta\Pi^{\sigma-1})(1-\theta\Pi^{\sigma})}.\end{aligned}$$

This expression is strictly positive for $\Pi\in(1,\theta^{-1/\sigma})$, including D itself (which is a function of Π per equation (23)). Thus, D is strictly increasing in Π for $\Pi\in[1,\theta^{-1/\sigma})$.

Consider next the labor share μ. Raising equation (23) to the power of $1+\psi$ and substituting in equation (24) yields

$$\begin{aligned}\mu &= \frac{\sigma-1}{\sigma(1+\tau)}\frac{1-\theta\Pi^{\sigma-1}}{1-\theta\Pi^{\sigma}}\frac{1-\beta\theta\Pi^{\sigma}}{1-\beta\theta\Pi^{\sigma-1}}\\ &= \frac{\sigma-1}{\sigma(1+\tau)}\left[1+\frac{(1-\beta)\theta\Pi^{\sigma-1}(\Pi-1)}{(1-\theta\Pi^{\sigma})(1-\beta\theta\Pi^{\sigma-1})}\right].\end{aligned}$$

Note that the fraction inside the brackets is strictly positive for $\Pi\in(1,\theta^{-1/\sigma})$ and is equal to zero for $\Pi=1$. Thus, $\mu\geq(\sigma-1)/[\sigma(1+\tau)]$, with equality only when $\Pi=1$. Differentiating this equation yields

$$\frac{\partial}{\partial\Pi}\mu = \left[\mu-\frac{\sigma-1}{\sigma(1+\tau)}\right]\left[\frac{\sigma-1}{\Pi}+\frac{1}{\Pi-1}+\frac{\sigma\theta\Pi^{\sigma-1}}{1-\theta\Pi^{\sigma}}+\frac{(\sigma-1)\beta\theta\Pi^{\sigma-2}}{1-\beta\theta\Pi^{\sigma-1}}\right].$$

This expression is strictly positive for $\Pi\in(1,\theta^{-1/\sigma})$. Thus, μ is strictly increasing in Π for $\Pi\in[1,\theta^{-1/\sigma})$.

B.3 Proof of Proposition 1

Below, we first consider the recursive formulation of the central bank's commitment problem presented in the main text. We then show that any steady state that satisfies the first-order conditions and Envelope conditions of this problem must feature $\Pi = 1$.

Statement of the Problem. In the sequential problem stated in the main text, at any given date t, the central bank with commitment takes price dispersion D_t and the committed values of the auxiliary variable δ_t and inflation Π_t as given. The central bank's problem can then be casted in the following recursive form, where given the values of D_t, δ_t and Π_t, the central bank chooses a labor share μ_t taking into account how that will translate to commitments to future values of price dispersion, inflation, and the auxiliary variable under the optimal behavior of the private sector:

$$\max_{(\mu_t,\Pi_{t+1},D_{t+1},\delta_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(-\log(D_t) + \frac{\log(\mu_t) - \mu_t}{1+\psi}\right)$$

subject to

$$D_{t+1} = (1-\theta)\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_{t+1}^{\sigma} D_t, \qquad (\beta^t\lambda_t)$$

$$\delta_t^{-1} = 1 + \beta\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-1}, \qquad (\beta^t\phi_t)$$

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \gamma\delta_t\mu_t D_t^{-1} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}, \qquad (\beta^t\kappa_t)$$

where $\gamma \equiv \frac{(1+\tau)\sigma}{\sigma-1}$, and λ, ϕ and κ are the assigned Lagrange multipliers to each of the corresponding constraints.

First-Order Conditions. For any $t \geq 0$, the first-order conditions of the central bank's problem above are as follows:

$$\mu_t : \quad \frac{\mu_t^{-1} - 1}{1+\psi} + \gamma\kappa_t\delta_t D_t^{-1} = 0 \iff \gamma\kappa_t\delta_t D_t^{-1} = \frac{1-\mu_t^{-1}}{1+\psi},$$

$$\delta_{t+1}: \quad -\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-2}\phi_t + \delta_{t+1}^{-2}\phi_{t+1} + \left[\gamma\mu_{t+1}D_{t+1}^{-1} - \Pi_{t+2}\left(\frac{1-\theta\Pi_{t+2}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}\right]\kappa_{t+1} = 0,$$

$$D_{t+1}: \quad -\beta D_{t+1}^{-1} - \lambda_t + \beta\theta\Pi_{t+2}^{\sigma}\lambda_{t+1} - \beta\gamma\delta_{t+1}\mu_{t+1}D_{t+1}^{-2}\kappa_{t+1} = 0,$$

$$\Pi_{t+1}: \quad \sigma\theta\Pi_{t+1}^{\sigma-2}\lambda_t\left[-\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{\sigma-1}} + D_t\Pi_{t+1}\right]$$

$$+\beta\theta(\sigma-1)\Pi_{t+1}^{\sigma-2}\delta_{t+1}^{-1}\phi_{t+1} - \beta\kappa_{t+1}\left(\frac{\theta\Pi_{t+1}^{\sigma-2}}{1-\theta\Pi_{t+1}^{\sigma-1}}\right)\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}$$

$$+\kappa_t(1-\delta_t)\frac{1}{1-\theta\Pi_{t+1}^{\sigma-1}}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = 0.$$

Steady State. In the steady state, the constraints become

$$D = \frac{1-\theta}{1-\theta\Pi^{\sigma}}\left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}},$$

$$\delta = 1-\beta\theta\Pi^{\sigma-1},$$

$$\left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{1-\beta\theta\Pi^{\sigma-1}}{1-\beta\theta\Pi^{\sigma}}\gamma\mu D^{-1}$$

$$\implies (\gamma\mu)^{-1} = \frac{1-\beta\theta\Pi^{\sigma-1}}{1-\beta\theta\Pi^{\sigma}}\frac{1-\theta\Pi^{\sigma}}{1-\theta\Pi^{\sigma-1}}.$$

The first-order conditions become

$$\mu: \quad \gamma\kappa\delta D^{-1} = \frac{1-\mu^{-1}}{1+\psi},$$

$$\delta: \quad \delta^{-2}\phi = \frac{1}{1-\theta\Pi^{\sigma-1}}\left[\Pi\left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} - \gamma\mu D^{-1}\right]\kappa,$$

$$D: \quad \lambda D = -\frac{\beta+\gamma\beta\delta\mu D^{-1}\kappa}{1-\beta\theta\Pi^{\sigma}},$$

$$\Pi: \quad \sigma\theta\Pi^{\sigma-2}\lambda\left[-\left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{1}{\sigma-1}} + D\Pi\right] + \beta\theta(\sigma-1)\Pi^{\sigma-2}\delta^{-1}\phi$$

$$+\left(\frac{1-\delta-\beta\theta\Pi^{\sigma-2}}{1-\theta\Pi^{\sigma-1}}\right)\left(\frac{1-\theta\Pi^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}\kappa = 0.$$

Step by Step Characterization. To prove the proposition, we proceed in steps as follows:

1. We use the first-order conditions for δ and μ in the steady state to obtain

$$\delta^{-2}\phi = \frac{1}{1-\theta\Pi^{\sigma-1}}\left(\frac{\Pi-1}{1-\beta\theta\Pi^{\sigma}}\right)\gamma\mu D^{-1}\kappa$$
$$\implies \delta^{-1}\phi = \frac{1}{1-\theta\Pi^{\sigma-1}}\left(\frac{\Pi-1}{1-\beta\theta\Pi^{\sigma}}\right)\frac{\mu-1}{1+\psi}.$$

2. We use the first-order conditions for D and μ in the steady state to obtain

$$\lambda D = -\frac{\beta}{1-\beta\theta\Pi^{\sigma}}\frac{\mu+\psi}{1+\psi}.$$

3. Consider the first-order condition for Π. Substituting with the expressions from steps 1 and 2 as well as the steady-state values of the constraints, we obtain

$$\begin{aligned}&-\sigma\beta\theta\Pi^{\sigma-2}\frac{1}{1-\beta\theta\Pi^{\sigma}}\frac{\mu+\psi}{1+\psi}\left(\frac{\Pi-1}{1-\theta\Pi^{\sigma-1}}\right)\\&+\beta\theta(\sigma-1)\Pi^{\sigma-2}\left(\frac{\Pi-1}{1-\theta\Pi^{\sigma-1}}\right)\frac{1}{1-\beta\theta\Pi^{\sigma}}\frac{\mu-1}{1+\psi}\\&+\beta\theta\Pi^{\sigma-2}\left(\frac{\Pi-1}{1-\theta\Pi^{\sigma-1}}\right)\frac{1}{1-\beta\theta\Pi^{\sigma}}\frac{\mu-1}{1+\psi}=0.\end{aligned}$$

4. Factoring out the common terms, the equation from step 3 yields

$$\sigma\frac{\beta\theta\Pi^{\sigma-2}}{1-\beta\theta\Pi^{\sigma}}\left(\frac{\Pi-1}{1-\theta\Pi^{\sigma-1}}\right)=0.$$

   It follows that the only possible steady-state value for inflation that respects the positivity of prices is $\Pi = 1$.

B.4 Proof of Proposition 2

Uniqueness. In the steady-state, $\dot{D}_t = \dot{\pi}_t = \dot{\delta}_t = 0$. Setting these to zero and dropping the time subscript, we obtain the following system of equations:

$$(\delta - \pi)[\lambda - (\sigma - 1)\pi] = \lambda \frac{\sigma(1+\tau)}{\sigma - 1} \left(1 - \frac{\sigma - 1}{\lambda}\pi\right)^{\frac{\sigma}{\sigma-1}} \frac{\delta}{D}, \tag{B.6}$$

$$(\lambda - \sigma\pi)D = \lambda \left(1 - \frac{\sigma - 1}{\lambda}\pi\right)^{\frac{\sigma}{\sigma-1}}, \tag{B.7}$$

$$\delta = \rho + \lambda - (\sigma - 1)\pi. \tag{B.8}$$

Substituting the last two equations into the first one yields

$$(\rho + \lambda - \sigma\pi)[\lambda - (\sigma - 1)\pi] = \frac{\sigma(1+\tau)}{\sigma - 1}(\lambda - \sigma\pi)[\rho + \lambda - (\sigma - 1)\pi],$$

which can be rewritten as

$$\frac{\rho(\sigma - 1)}{1 + \sigma\tau}\pi = (\lambda - \sigma\pi)[\rho + \lambda - (\sigma - 1)\pi]. \tag{B.9}$$

Since this is a quadratic equation, there are at most two steady-state values of π that solve it. Rather than solving for these roots explicitly, observe that the left-hand side of the equation is a linear increasing function of π, while the right-hand side has two zeros, one at $\pi = \frac{\lambda}{\sigma}$ and another at $\pi = \frac{\rho+\lambda}{\sigma-1}$. Since $\frac{\lambda}{\sigma} < \frac{\rho+\lambda}{\sigma-1}$, we need to consider three regions:

1. $\pi < \frac{\lambda}{\sigma}$: In this region, the right-hand side of (B.9) is positive. The two sides intersect at a point where both are positive, so the quadratic has at least one root $\pi \in (0, \frac{\lambda}{\sigma})$.
2. $\frac{\lambda}{\sigma} \leq \pi \leq \frac{\rho+\lambda}{\sigma-1}$: In this region, the right-hand side of (B.9) is negative while the left-hand side is strictly positive. Thus, there cannot be a solution here.
3. $\pi > \frac{\rho+\lambda}{\sigma-1}$: In this region, the right-hand side of (B.9) is positive and grows quadratically from 0, whereas the left-hand side grows linearly from a positive number. The two sides intersect at a point where both

are positive, so the quadratic has at least one root $\pi \in (\frac{\rho+\lambda}{\sigma-1}, \infty)$. Since a quadratic cannot have more than two roots, we conclude that the roots found in the first and third regions above are unique within their regions.

Finally, note that the root $\pi > \frac{\rho+\lambda}{\sigma-1}$ violates the natural bound on inflation implied by sticky prices $\pi < \frac{\lambda}{\sigma-1}$ and thus cannot be a steady state. Therefore, the unique steady state is the one found in the first region, $\pi \in (0, \frac{\lambda}{\sigma})$.

Comparative Statics. It follows from the proof of uniqueness above that steady-state inflation $\pi_{ss}(\tau, \sigma)$ solves

$$\frac{\rho(\sigma-1)}{1+\sigma\tau}\pi_{ss}(\tau,\sigma) = (\lambda - \sigma\pi_{ss}(\tau,\sigma))[\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)], \tag{B.10}$$

where the value of $\pi_{ss}(\tau, \sigma)$ is the root of this quadratic equation in the interval $(0, \frac{\lambda}{\sigma})$. Given this value, we can then derive steady-state price dispersion $D_{ss}(\tau, \sigma)$ using equation (B.7):

$$D_{ss}(\tau,\sigma) = \frac{\lambda}{\lambda - \sigma\pi_{ss}(\tau,\sigma)}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}(\tau,\sigma)\right)^{\frac{\sigma}{\sigma-1}}. \tag{B.11}$$

Part 1. Consider first $\pi_{ss}(\tau, \sigma)$. Differentiating (B.10) with respect to τ yields

$$\left[\frac{\sigma}{\lambda-\sigma\pi_{ss}(\tau,\sigma)} + \frac{\sigma-1}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)} + \frac{1}{\pi_{ss}(\tau,\sigma)}\right]\frac{\partial}{\partial\tau}\pi_{ss}(\tau,\sigma) = \frac{\sigma}{1+\sigma\tau}.$$

All the terms in the bracket on the left-hand side are positive given $\pi_{ss}(\tau, \sigma) \in (0, \frac{\lambda}{\sigma})$. The right-hand side is also positive by Assumption 1. Thus, $\frac{\partial}{\partial\tau}\pi_{ss}(\tau, \sigma) > 0$ and $\pi_{ss}(\tau, \sigma)$ is strictly increasing in τ.

Consider next $D_{ss}(\tau, \sigma)$. From (B.11), we see that $D_{ss}(\tau, \sigma)$ depends on τ only through $\pi_{ss}(\tau, \sigma)$. Thus,

$$\begin{aligned}\frac{\partial}{\partial\tau}D_{ss}(\tau,\sigma) &= \frac{\partial}{\partial\pi_{ss}(\tau,\sigma)}D_{ss}(\tau,\sigma) \times \frac{\partial}{\partial\tau}\pi_{ss}(\tau,\sigma)\\ &= \frac{\sigma D_{ss}(\tau,\sigma)\pi_{ss}(\tau,\sigma)}{(\lambda-\sigma\pi_{ss}(\tau,\sigma))[\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)]}\frac{\partial}{\partial\tau}\pi_{ss}(\tau,\sigma).\end{aligned}$$

All the terms involved are positive given $\pi_{ss}(\tau,\sigma) \in (0, \frac{\lambda}{\sigma})$. Thus, $\frac{\partial}{\partial \tau} D_{ss}(\tau,\sigma) > 0$ and $D_{ss}(\tau,\sigma)$ is strictly increasing in τ.

Part 2. Consider first $\pi_{ss}(\tau,\sigma)$. Differentiating (B.10) with respect to σ yields

$$\left[\frac{\sigma-1}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}+\frac{\sigma}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}+\frac{1}{\pi_{ss}(\tau,\sigma)}\right]\frac{\partial}{\partial\sigma}\pi_{ss}(\tau,\sigma)$$
$$=-\left[\frac{1+\tau}{(\sigma-1)(1+\sigma\tau)}+\frac{\pi_{ss}(\tau,\sigma)}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}+\frac{\pi_{ss}(\tau,\sigma)}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}\right]. \tag{B.12}$$

Using $\pi_{ss}(\tau,\sigma) \in (0, \frac{\lambda}{\sigma})$ and Assumption 1, we can conclude that all the terms inside the brackets on both sides are positive. Thus, by the negative sign on the right-hand side, $\frac{\partial}{\partial\sigma}\pi_{ss}(\tau,\sigma) < 0$ and $\pi_{ss}(\tau,\sigma)$ is strictly decreasing in σ.

Consider next $D_{ss}(\tau,\sigma)$. Observe that $D_{ss}(\tau,\sigma)$ depends on σ both directly through aggregation, and indirectly through $\pi_{ss}(\tau,\sigma)$ as the central bank's optimal policy changes $\pi_{ss}(\sigma,\tau)$ when σ varies. Accordingly, we will investigate the total derivative of $D_{ss}(\tau,\sigma)$ by decomposing it into these direct and indirect effects of σ:

$$\frac{\partial}{\partial\sigma}D_{ss}(\tau,\sigma)=\frac{\partial}{\partial\sigma}D_{ss}(\tau,\sigma)\big|_{\pi_{ss}(\tau,\sigma)}+\frac{\partial}{\partial\pi_{ss}(\tau,\sigma)}D_{ss}(\tau,\sigma)\big|_{\sigma}\times\frac{\partial}{\partial\sigma}\pi_{ss}(\tau,\sigma). \tag{B.13}$$

To derive the first term on the right-hand side, we use (B.11) to obtain

$$\begin{aligned}\frac{\partial}{\partial\sigma}D_{ss}(\tau,\sigma)\big|_{\pi_{ss}(\tau,\sigma)}=&\frac{D_{ss}(\tau,\sigma)}{(\sigma-1)^2}\left(1-\frac{1}{1-\frac{\sigma-1}{\lambda}\pi_{ss}(\tau,\sigma)}-\log\left(1-\frac{\sigma-1}{\lambda}\pi_{ss}(\tau,\sigma)\right)\right)\\&+D_{ss}(\tau,\sigma)\left[\frac{\pi_{ss}(\tau,\sigma)}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}-\frac{\pi_{ss}(\tau,\sigma)}{\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}\right].\end{aligned}$$

As for the partial derivative of $D_{ss}(\tau,\sigma)$ with respect to $\pi_{ss}(\tau,\sigma)$, holding σ fixed, we use (B.11) to obtain

$$\frac{\partial}{\partial\pi_{ss}(\tau,\sigma)}D_{ss}(\tau,\sigma)\big|_{\sigma}=D_{ss}(\tau,\sigma)\left[\frac{\sigma}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}-\frac{\sigma}{\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}\right].$$

Substituting these into (B.13) yields

$$\frac{\partial}{\partial\sigma}D_{ss}(\tau,\sigma) = \frac{D_{ss}(\tau,\sigma)}{(\sigma-1)^2}\overbrace{\left(1-\frac{1}{1-\frac{\sigma-1}{\lambda}\pi_{ss}(\tau,\sigma)}-\log\left(1-\frac{\sigma-1}{\lambda}\pi_{ss}(\tau,\sigma)\right)\right)}^{\textcircled{1}<0}$$
$$+D_{ss}(\tau,\sigma)\underbrace{\left(\sigma\frac{\partial}{\partial\sigma}\pi_{ss}(\tau,\sigma)+\pi_{ss}(\tau,\sigma)\right)}_{\textcircled{2}}\underbrace{\left[\frac{1}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}-\frac{1}{\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}\right]}_{\textcircled{3}>0}.$$

It is straightforward to show that ① is strictly negative for $\pi_{ss}(\tau,\sigma)\in(0,\frac{\lambda}{\sigma})$.[39] Moreover, ③ is strictly positive for $\pi_{ss}(\tau,\sigma)\in(0,\frac{\lambda}{\sigma})$. Thus, a sufficient condition for $\frac{\partial}{\partial\sigma}D_{ss}(\tau,\sigma)$ to be strictly negative is that ② is negative. We next show that this holds under $\tau<\bar{\tau}(\sigma)$. Using (B.12), we have

$$\textcircled{2} = -\frac{\frac{\sigma(1+\tau)}{(\sigma-1)(1+\sigma\tau)}+\frac{\sigma\pi_{ss}(\tau,\sigma)}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}+\frac{\sigma\pi_{ss}(\tau,\sigma)}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}}{\frac{\sigma-1}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}+\frac{\sigma}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}+\frac{1}{\pi_{ss}(\tau,\sigma)}}+\pi_{ss}(\tau,\sigma)$$

$$=\frac{-\frac{\sigma(1+\tau)}{(\sigma-1)(1+\sigma\tau)}-\frac{\pi_{ss}(\tau,\sigma)}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}+1}{\frac{\sigma-1}{\rho+\lambda-(\sigma-1)\pi_{ss}(\tau,\sigma)}+\frac{\sigma}{\lambda-\sigma\pi_{ss}(\tau,\sigma)}+\frac{1}{\pi_{ss}(\tau,\sigma)}}.$$

The denominator is positive for $\pi_{ss}(\tau,\sigma)$ in $(0,\frac{\lambda}{\sigma})$. We show that the numerator is negative for $\tau<\bar{\tau}(\sigma)$. To see this, note that the fraction involving $\pi_{ss}(\tau,\sigma)$ is negative, so it is sufficient to show that

$$-\frac{\sigma(1+\tau)}{(\sigma-1)(1+\sigma\tau)}+1<0 \iff (\sigma-2)\sigma\tau<1.$$

[39] To see this, note that $\pi_{ss}(\tau,\sigma)\in(0,\frac{\lambda}{\sigma})$ implies that $1-\frac{\sigma-1}{\lambda}\pi_{ss}(\tau,\sigma)\in(\frac{1}{\sigma},1)$. Moreover, note that the function $f(x)\equiv 1-1/x-\log(x)$ is strictly increasing in $x\in(0,1)$ (as $f'(x)=1/x^2-1/x>0, x\in(0,1)$), so that $\forall x\in(\frac{1}{\sigma},1): f(x)<f(1)=0$.

Now note that under $\tau < \bar{\tau}(\sigma)$ and Assumption 1, we have

$$1 < \sigma < 2 \implies (\sigma - 2)\sigma\tau < (2 - \sigma) < 1,$$
$$\sigma \geq 2 \implies (\sigma - 2)\sigma\tau < (\sigma - 2)\sigma\bar{\tau}(\sigma) = (\sigma - 2)\sigma\frac{1}{\sigma(\sigma - 2)} = 1.$$

Hence, given $\tau < \bar{\tau}(\sigma)$ and $\sigma > 1$, we obtain ② < 0. It follows that $\frac{\partial}{\partial\sigma}D_{ss}(\tau, \sigma) < 0$ and $D_{ss}(\tau, \sigma)$ is strictly decreasing in σ for all $\tau < \bar{\tau}(\sigma)$.

B.5 Proof of Proposition 3

To prove this proposition, we will rely on the Stable Manifold and the Hartman-Grobman theorems (Perko, 2001, pages 107 and 120, respectively). These two theorems relate the dynamics of a non-linear dynamical system to its local linearized dynamics around a fixed point (in our case, the unique steady state). To make use of their predictions, we rewrite our dynamical system involving the variables π_t, D_t and δ_t in the following form. Let $X_t \equiv (\pi_t, D_t, \delta_t)$. Then the non-linear dynamical system implied by the model can be characterized by a function $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined as

$$\dot{X}_t = f(X_t) \equiv \begin{bmatrix} -\lambda^{\frac{\sigma(1+\tau)}{\sigma-1}} \left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} \frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma - 1)\pi_t] \\ \lambda \left(1 - \frac{\sigma-1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma\pi_t - \lambda)D_t \\ \delta_t^2 + [(\sigma - 1)\pi_t - (\rho + \lambda)]\delta_t \end{bmatrix},$$

where the unique steady state that we characterized is a fixed point of this system.

Note that $f(\cdot)$ is a smooth function; importantly, it is continuously differentiable, which implies that the flows of the system are also continuous. In order to understand the dynamics of the system and how the transition to a new steady state happens, we need to first characterize the nature of the unique steady state for the above system. To do this, we can apply the Hartman-Grobman theorem, which states that if the eigenvalues of the Jacobian of the function f evaluated at the fixed point have non-zero real parts, then there exists a neighborhood N around the fixed point of the system where the flows of the non-linear system are topologically conjugate to the flows of the linearized system. We will apply this theorem in the following way. First, we will show that the fixed point is a saddle point of the linearized system. Then verifying the assumptions of the Hartman-Grobman theorem, we will conclude from topological conjugacy that the steady state is also a saddle point of the non-linear system.

To show that the steady state is a saddle point of the linearized system, we first need to compute the Jacobian of f at the steady state. Letting $X_{ss} = (\pi_{ss}, D_{ss}, \delta_{ss})$ denote the steady state under a certain set of parameters, note that

$$0 = \dot{X}_{ss} = f(X_{ss}) \implies \begin{cases} \frac{\rho(\sigma-1)}{1+\sigma\tau}\pi_{ss} = (\lambda - \sigma\pi_{ss})[\rho + \lambda - (\sigma-1)\pi_{ss}] \\ D_{ss} = \frac{\lambda}{\lambda-\sigma\pi_{ss}}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{\sigma}{\sigma-1}} \\ \delta_{ss} = \rho + \lambda - (\sigma-1)\pi_{ss} \end{cases}, \tag{B.14}$$

and, letting $\mathbf{D}f$ denote the Jacobian of f evaluated at X_{ss}, we have

$$\mathbf{D}f = \begin{bmatrix} \frac{\partial}{\partial\pi}f_1 & \frac{\partial}{\partial D}f_1 & \frac{\partial}{\partial\delta}f_1 \\ \frac{\partial}{\partial\pi}f_2 & \frac{\partial}{\partial D}f_2 & \frac{\partial}{\partial\delta}f_2 \\ \frac{\partial}{\partial\pi}f_3 & \frac{\partial}{\partial D}f_3 & \frac{\partial}{\partial\delta}f_3 \end{bmatrix},$$

where all the partial derivatives are evaluated at X_{ss} and are given by

$$\begin{aligned} \frac{\partial}{\partial\pi}f_1 &= \frac{\sigma^2(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{1}{\sigma-1}}\frac{\delta_{ss}}{D_{ss}} - [\lambda - (\sigma-1)\pi_{ss}] - (\sigma-1)(\delta_{ss} - \pi_{ss}) \\ &= \rho - \pi_{ss}, \qquad \text{(using equation (B.14))} \\ \frac{\partial}{\partial D}f_1 &= \lambda\frac{\sigma(1+\tau)}{\sigma-1}\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_{ss}}{D_{ss}^2} = \frac{\rho\pi_{ss} + (\lambda-\sigma\pi_{ss})\delta_{ss}}{D_{ss}}, \\ \frac{\partial}{\partial\delta}f_1 &= -\frac{1+\sigma\tau}{\sigma-1}(\lambda - \sigma\pi_{ss}) + \pi_{ss} = \frac{\pi_{ss}}{\delta_{ss}}[\lambda - (\sigma-1)\pi_{ss}], \\ \frac{\partial}{\partial\pi}f_2 &= -\sigma\left(1 - \frac{\sigma-1}{\lambda}\pi_{ss}\right)^{\frac{1}{\sigma-1}} + \sigma D_{ss} = \sigma D_{ss}\frac{\pi_{ss}}{\lambda-(\sigma-1)\pi_{ss}}, \end{aligned}$$

$$\frac{\partial}{\partial D} f_2 = \sigma \pi_{ss} - \lambda,$$
$$\frac{\partial}{\partial \delta} f_2 = 0,$$
$$\frac{\partial}{\partial \pi} f_3 = (\sigma - 1)\delta_{ss},$$
$$\frac{\partial}{\partial D} f_3 = 0,$$
$$\frac{\partial}{\partial \delta} f_3 = 2\delta_{ss} + (\sigma - 1)\pi_{ss} - (\rho + \lambda) = \delta_{ss}.$$

To show that the Hartman-Grobman theorem applies, we need to show that X_{ss} is a hyperbolic fixed point—i.e., all the eigenvalues of $\mathbf{D}f$ have non-zero real parts. To calculate the eigenvalues of $\mathbf{D}f$, we need to compute the roots of its characteristic polynomial:

$$\det(\mathbf{D}f - \eta \mathbf{I}) = 0,$$

where any η that solves this polynomial is an eigenvalue of the Jacobian. The characteristic polynomial is given by:

$$\det(\mathbf{D}f - \eta \mathbf{I}) =$$
$$(\tfrac{\partial}{\partial \pi} f_1 - \eta)(\tfrac{\partial}{\partial D} f_2 - \eta)(\tfrac{\partial}{\partial \delta} f_3 - \eta) - \tfrac{\partial}{\partial D} f_1 \tfrac{\partial}{\partial \pi} f_2 \left(\tfrac{\partial}{\partial \delta} f_3 - \eta\right) - \tfrac{\partial}{\partial \delta} f_1 \tfrac{\partial}{\partial \pi} f_3 \left(\tfrac{\partial}{\partial D} f_2 - \eta\right),$$

where we have used $\frac{\partial}{\partial \delta} f_2 = \frac{\partial}{\partial D} f_3 = 0$. Plugging in the derived values for other partial derivatives, we obtain the following cubic polynomial:

$$\begin{aligned}
&\det(\mathbf{D}f - \eta \mathbf{I}) \\
&= (\rho - \pi_{ss} - \eta)(\sigma \pi_{ss} - \lambda - \eta)(\delta_{ss} - \eta) - \sigma \pi_{ss}(\rho + \lambda - \sigma \pi_{ss})(\delta_{ss} - \eta) \\
&- (\sigma - 1)\pi_{ss}[\lambda - (\sigma - 1)\pi_{ss}](\sigma \pi_{ss} - \lambda - \eta).
\end{aligned}$$

We now need to compute the roots of this cubic equation. One could use the general formula for roots of a cubic but that requires some tedious algebra. An easier path is to guess and verify that one of the roots is ρ.[40] To verify this

[40] There is an economic intuition for this guess. We know that at $\rho = 0$, the Phillips curve of the economy is fully vertical, which implies that $\rho = 0$ is a bifurcation point for the system. So the behavior of the system should switch at $\rho = 0$, making it reasonable to guess that ρ is one of its eigenvalues.

guess observe that at $\eta = \rho$,

$$\det\left(\mathbf{D}f - \rho\mathbf{I}\right) = (\sigma-1)\pi_{ss}(\sigma\pi_{ss}-\lambda-\rho)(\delta_{ss}-\rho) - (\sigma-1)\pi_{ss}(\delta_{ss}-\rho)(\sigma\pi_{ss}-\lambda-\rho) = 0.$$

Thus, the characteristic polynomial is divisible by $\rho - \eta$. Using this fact, we can factorize the characteristic polynomial as

$$\det(\mathbf{D}f - \eta\mathbf{I}) = (\rho - \eta)\left[\eta^2 - \rho\eta - (\rho+\lambda)\lambda + \sigma(\sigma-1)\pi_{ss}^2\right],$$

where the rest of the eigenvalues are the roots of the quadratic equation $\eta^2 - \rho\eta - (\rho+\lambda)\lambda + \sigma(\sigma-1)\pi_{ss}^2 = 0$. Therefore, the eigenvalues of the Jacobian at the steady state are

$$\eta = \begin{cases} \eta_1 \equiv \rho \\ \eta_2 \equiv \frac{\rho}{2} + \sqrt{\left(\frac{\rho}{2}+\lambda\right)^2 - \sigma(\sigma-1)\pi_{ss}^2} \\ \eta_3 \equiv \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}+\lambda\right)^2 - \sigma(\sigma-1)\pi_{ss}^2} \end{cases}.$$

We can make the following observations about these eigenvalues. First, all of them are real. To see this, we just need to confirm that the term inside the square root is always positive. This follows from $\rho > 0$ and the fact that $\pi_{ss} \in (0, \lambda/\sigma)$ under Assumption 1:

$$\left(\frac{\rho}{2}+\lambda\right)^2 - \sigma(\sigma-1)\pi_{ss}^2 > \lambda^2 - \sigma^2\pi_{ss}^2 = (\lambda - \sigma\pi_{ss})(\lambda + \sigma\pi_{ss}) > 0.$$

A second observation is that the first two eigenvalues are strictly positive (which is straightforward to confirm from the observation above) and the third one is negative. To verify the latter, note that

$$\begin{aligned} \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}+\lambda\right)^2 - \sigma(\sigma-1)\pi_{ss}^2} < 0 &\iff \left(\frac{\rho}{2}\right)^2 < \left(\frac{\rho}{2}+\lambda\right)^2 - \sigma(\sigma-1)\pi_{ss}^2 \\ &\iff 0 < \lambda^2 + \rho\lambda - \sigma(\sigma-1)\pi_{ss}^2, \end{aligned}$$

and the last inequality holds since

$$\lambda^2 + \rho\lambda - \sigma(\sigma-1)\pi_{ss}^2 > \lambda^2 - \sigma^2\pi_{ss}^2 = (\lambda - \sigma\pi_{ss})(\lambda + \sigma\pi_{ss}) > 0.$$

Therefore, the Jacobian $\mathbf{D}f$ has two strictly positive eigenvalues and one strictly negative eigenvalue. This implies that the fixed point X_{ss} is a hyperbolic fixed point and is a saddle point for the linearized dynamical system. Thus, the Hartman-Grobman theorem applies and we can conclude that the fixed point is also a saddle point for the non-linear system.

Since all eigenvalues are distinct, the three eigenvectors associated with them are linearly independent and span $\mathbb{R}^3$. Thus, these eigenvalues imply that the dynamics of the linearized system are stable along the eigenspace spanned by the negative eigenvalue (which is one-dimensional as we show below) and unstable along the eigenspace associated with the two positive eigenvalues. Now, to study the convergence of the non-linear dynamics, we appeal to the Stable Manifold Theorem. When applied to our setting, this theorem states that in an open neighborhood around the fixed point X_{ss} where the function f is continuously differentiable (which is the case for our system), there exists a one-dimensional differentiable manifold S tangent to the stable subspace of the linear system such that for all $t \geq 0$, $X \in S$,

$$\lim_{t\to\infty} \phi_t(X) = X_{ss},$$

where $\phi_t(X)$ denotes the flow of the non-linear system starting from X at time $t = 0$ (i.e., $\phi_0(X) = X$) and evolves according to the non-linear dynamics. Therefore, we have established that in an open neighborhood N of the fixed point X_{ss}, the non-linear dynamics converge to the fixed point X_{ss} along a stable manifold S that is one-dimensional and tangent to the one-dimensional eigenspace of the linearized system at the fixed point. It then suffices to characterize the direction of convergence along the stable eigenspace of the linearized system. To this end, consider the linear dynamics around the fixed point X_{ss}:

$$\dot{X}_t = \mathbf{D}f\left(X_t - X_{ss}\right).$$

Let $\psi_t(X)$ denote the flow of this linearized system starting from some $X \in \mathbb{R}^3$. Since the eigenvectors of $\mathbf{D}f$ are linearly independent, we can write this flow as

$$\psi_t(X) = \alpha_{1,X}(t)\mathbf{v}_1 + \alpha_{2,X}(t)\mathbf{v}_2 + \alpha_{3,X}(t)\mathbf{v}_3,$$

where $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are eigenvectors of $\mathbf{D}f$ that correspond to eigenvalues η_1, η_2, and η_3 respectively. Furthermore, since $\psi_0(X) = X$, $\alpha_{i,X}(0)$ for $i = 1, 2, 3$ are given by the projection of X on the eigenvectors of $\mathbf{D}f$. Also, note that since $\psi_t(X_{ss}) = X_{ss}$, $\alpha_{i,X_{ss}}(t)$ is constant over time, and we use $\bar{\alpha}_i$ to refer to it. Plugging this decomposition into the linearized system yields

$$\sum_{i=1}^{3} \dot{\alpha}_{i,X}(t)\mathbf{v}_i = \mathbf{D}f \sum_{i=1}^{3}(\alpha_{i,X}(t) - \bar{\alpha}_i)\mathbf{v}_i = \sum_{i=1}^{3} \eta_i(\alpha_{i,X}(t) - \bar{\alpha}_i)\mathbf{v}_i.$$

Therefore, for $i = 1, 2, 3$,

$$\dot{\alpha}_{i,X}(t) = \eta_i(\alpha_{i,X}(t) - \bar{\alpha}_i) \implies \alpha_{i,X}(t) - \bar{\alpha}_i = (\alpha_{i,X}(0) - \bar{\alpha}_i)e^{\eta_i t},$$

which implies

$$\psi_t(X) = X_{ss} + \sum_{i=1}^{3}(\alpha_{i,X}(0) - \bar{\alpha}_i)e^{\eta_i t}\mathbf{v}_i.$$

Note that since the $\mathbf{v}_i$'s are linearly independent, $\psi_t(X)$ is convergent if and only if $\alpha_{1,X}(0) - \bar{\alpha}_1 = \alpha_{2,X}(0) - \bar{\alpha}_2 = 0$ (since $\eta_1 > 0$ and $\eta_2 > 0$). This identifies the stable eigenspace of the linearized system as the span of $\mathbf{v}_3$ shifted to cross X_{ss}; that is,

$$\begin{aligned}\lim_{t\to\infty} \psi_t(X) = X_{ss} &\iff X \in X_{ss} + \operatorname{span}(\mathbf{v}_3) \\ &\iff \psi_t(X) - X_{ss} = k e^{\eta_3 t}\mathbf{v}_3 \quad \text{for some } k \in \mathbb{R}.\end{aligned}$$

Given that $\mathbf{v}_3 = (v_{3,1}, v_{3,2}, v_{3,3})$ is an eigenvector associated with the negative eigenvalue η_3, and normalizing $v_{3,1} = 1$, we have

$$\frac{\partial}{\partial \pi} f_2 + \left(\frac{\partial}{\partial D} f_2 - \eta_3\right) v_{3,2} = 0 \implies v_{3,2} = \frac{\frac{\partial}{\partial \pi} f_2}{\eta_3 - \frac{\partial}{\partial D} f_2},$$
$$\frac{\partial}{\partial \pi} f_3 + \left(\frac{\partial}{\partial \delta} f_3 - \eta_3\right) v_{3,3} = 0 \implies v_{3,3} = \frac{\frac{\partial}{\partial \pi} f_3}{\eta_3 - \frac{\partial}{\partial \delta} f_3}.$$

For a given $k \in \mathbb{R}$, let $\psi_t(X) - X_{ss} = (D_t^L - D_{ss}, \pi_t^L - \pi_{ss}, \delta_t^L - \delta_{ss})$ denote the flow of the linearized system towards the steady state. We show that along the transition path, if D_t^L converges to D_{ss} from below, then π_t^L converges to π_{ss} from above and vice versa. To see this, note that

$$\frac{\pi_t^L - \pi_{ss}}{D_t^L - D_{ss}} = \frac{v_{3,1}}{v_{3,2}} = \frac{\eta_3 - \frac{\partial}{\partial D} f_2}{\frac{\partial}{\partial \pi} f_2} = \frac{\eta_3 - \sigma \pi_{ss} + \lambda}{\sigma D_{ss} \pi_{ss}} [\lambda - (\sigma - 1)\pi_{ss}].$$

In the expression above, $\sigma D_{ss} \pi_{ss} > 0$ and $\lambda - (\sigma - 1)\pi_{ss} > 0$ as $\pi_{ss} \in (0, \lambda/\sigma)$. Thus, to conclude that the ratio has a negative sign, we need to show that $\eta_3 - \sigma \pi_{ss} + \lambda < 0$. To see that this is indeed the case, note that

$$\begin{aligned}
\eta_3 + \lambda - \sigma \pi_{ss} < 0 &\iff \lambda - \sigma \pi_{ss} + \frac{\rho}{2} < \sqrt{\left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2} \\
&\iff \left(\frac{\rho}{2} + \lambda\right)^2 + \sigma^2 \pi_{ss}^2 - (2\lambda + \rho)\sigma \pi_{ss} < \left(\frac{\rho}{2} + \lambda\right)^2 - \sigma(\sigma - 1)\pi_{ss}^2 \\
&\iff 2\sigma \pi_{ss} - 2\lambda - \rho - \pi_{ss} < 0,
\end{aligned}$$

and the last inequality holds since $\pi_{ss} \in (0, \lambda/\sigma)$. Hence, linearized dynamics are such that

$$\kappa \equiv \frac{\pi_t^L - \pi_{ss}}{D_t^L - D_{ss}} < 0.$$

Finally, let $\phi_t(X) - X_{ss} = (D_t - D_{ss}, \pi_t - \pi_{ss}, \delta_t - \delta_{ss})$ denote the flow of the non-linear system starting from an X on the one-dimensional stable manifold so that $\lim_{t \to \infty} \phi_t(X) = X_{ss}$. Since the stable manifold is tangent to the stable subspace of the linearized system, for sufficiently small $\varepsilon > 0$ such that

$\varepsilon + \kappa < 0$, there exists $\bar{t} \geq 0$ such that for all $t > \bar{t}$,

$$\frac{\pi_t - \pi_{ss}}{D_t - D_{ss}} \in (\kappa - \varepsilon, \kappa + \varepsilon) \implies \frac{\pi_t - \pi_{ss}}{D_t - D_{ss}} < 0.$$

Hence, there exists $\bar{t} \geq 0$ such that, after time $\bar{t}$, if D_t of the non-linear system converges to D_{ss} from below, then π_t of the non-linear system converges to π_{ss} from above and vice versa.

To conclude the proof of Proposition 3, consider a change in the parameters of the model that leads to an increase in D_{ss}, as is the case in both parts 1 and 2 of the proposition. First note that since our non-linear system is continuously differentiable, D_t (along with π_t and δ_t) have continuous paths along the transition. Moreover, since D_t is backward-looking, it is also continuous at $t = 0$ (i.e., $\lim_{t \to 0} D_t = D_0$, unlike π_t and δ_t which jump to the stable manifold to accommodate convergence to the steady state). Thus, it has to be that conditional on converging to the new steady state, D_t is a continuous function of time with $D_0 < D_{ss} = \lim_{t \to \infty} D_t$.

If along the transition path D_t never crosses D_{ss}, then $D_t - D_{ss} < 0$ for all t. This means that there exists $\bar{t} \geq 0$ such that $\pi_t - \pi_{ss} > 0$ for all $t > \bar{t}$.

Suppose instead that D_t crosses D_{ss} along the transition path to possibly converge to D_{ss} from above. If this was possible, then there would be two paths for convergence starting from D_{ss}: one that increases and then converges back to D_{ss} from above, and another that starts at D_{ss} and stays at D_{ss} forever. However, in this case, the equilibrium cannot be Markov. Therefore, the only possibility of convergence in a Markov equilibrium is that D_t converges to D_{ss} from below, and thus π_t converges to π_{ss} from above.

C Derivations for the Limit Setting with $\sigma \to 1$

In Section 5.3, we considered a special case of our model that takes the elasticity of substitution $\sigma \to 1$, with the labor wedge τ adjusting so as to keep monopoly $\gamma \equiv \frac{\sigma(1+\tau)}{\sigma-1}$ constant. Below, we provide the derivations for this limit setting.

Recall from equations (31) and (32) that in the continuous-time limit of our model, the dynamics of dispersion and inflation are given by

$$\dot{D}_t = \lambda \left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} + (\sigma\pi_t - \lambda)D_t, \tag{C.1}$$

$$\dot{\pi}_t = -\lambda\frac{\sigma(1+\tau)}{\sigma - 1}\left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} \frac{\delta_t}{D_t} + (\delta_t - \pi_t)[\lambda - (\sigma - 1)\pi_t], \tag{C.2}$$

where

$$\dot{\delta}_t = \delta_t^2 + [(\sigma - 1)\pi_t - (\rho + \lambda)]\delta_t. \tag{C.3}$$

Taking the limit of this system as $\sigma \to 1$ while keeping γ fixed, we arrive at

$$\dot{D}_t = \lambda e^{-\frac{\pi_t}{\lambda}} + (\pi_t - \lambda)D_t, \tag{C.4}$$

$$\dot{\pi}_t = -\lambda\gamma e^{-\frac{\pi_t}{\lambda}}\frac{\delta_t}{D_t} + (\delta_t - \pi_t)\lambda, \tag{C.5}$$

$$\dot{\delta}_t = \delta_t^2 - (\rho + \lambda)\delta_t. \tag{C.6}$$

Importantly, we observe from equation (C.6) that, in this limit, the dynamics of δ_t are decoupled from the rest of the system and given by a linear differential equation that involves only δ_t itself. The dynamics of δ_t for an arbitrary flow are thus given by the general solution to this differential equation:

$$\delta_t = \frac{\rho + \lambda}{1 + Ke^{(\rho+\lambda)t}}, \tag{C.7}$$

where K is a constant that indexes the flow of the system. We note that the only value of K that is consistent with converging to the steady state of the system ($\delta_{ss} = \rho + \lambda$) is $K = 0$, implying that $\delta_t = \rho + \lambda$ along the whole transition path. That is, δ_t jumps to its steady-state value immediately at $t = 0$ and stays there until the rest of the system converges. Plugging this into equations (C.4) and (C.5) yields

$$\dot{D}_t = \lambda e^{-\frac{\pi_t}{\lambda}} + (\pi_t - \lambda)\, D_t,$$

$$\dot{\pi}_t = -\lambda\left(\rho+\lambda\right)\gamma e^{-\frac{\pi_t}{\lambda}}\frac{1}{D_t} + \left(\rho+\lambda-\pi_t\right)\lambda.$$

This is still a system of two non-linear differential equations, but one that can be solved in closed form conditional on converging to the steady state. To see this, consider the following change of variables: let $d_t \equiv \log D_t$ and $x_t \equiv \frac{\pi_t}{\lambda} + d_t$. We then have

$$\begin{aligned}
\dot{d}_t &= \lambda e^{-x_t} + (\pi_t - \lambda), \\
\frac{\dot{\pi}_t}{\lambda} &= -(\rho+\lambda)\gamma e^{-x_t} + (\rho+\lambda-\pi_t).
\end{aligned}$$

Summing these two equations, we obtain

$$\dot{x}_t = -\left[\lambda(\gamma-1)+\rho\gamma\right]e^{-x_t} + \rho,$$

where $\lambda(\gamma-1)+\rho\gamma > 0$ under Assumption 1.[41] This is a univariate differential equation in terms of x_t that can be rearranged as

$$\begin{aligned}
(\dot{x}_t - \rho)e^{x_t} &= -\left[\lambda(\gamma-1)+\rho\gamma\right] \\
\Longleftrightarrow \frac{d}{dt}\left(e^{x_t-\rho t}\right) &= -\left[\lambda(\gamma-1)+\rho\gamma\right]e^{-\rho t},
\end{aligned}$$

yielding the general solution

$$e^{x_t} = \left[\gamma + (\gamma-1)\frac{\lambda}{\rho}\right] + K' e^{\rho t}$$

for some constant K'. We note that conditional on converging to the steady state, where $\dot{\pi}_t = \dot{d}_t = 0$, we must have $\dot{x}_t = 0$. Thus, the only flow for x_t that is consistent with converging to a steady state is when $K' = 0$, giving us the solution

$$x_t = \log\left(\gamma + (\gamma-1)\frac{\lambda}{\rho}\right), \forall t \geq 0$$

[41] Recall that for a given σ, Assumption 1 requires that $\tau > -1/\sigma$, which can be rewritten as $\gamma = \sigma(1+\tau)/(\sigma-1) > 1$. Since we took the limit of $\sigma \to 1$ holding γ fixed, this statement of Assumption 1 remains invariable to σ.

along the transition path. Substituting this into the original system, we observe

$$\begin{aligned}\dot{d}_t &= -\lambda\left[d_t + \frac{(\gamma-1)(\rho+\lambda)}{\rho\gamma+(\gamma-1)\lambda} - \log\left(\gamma+(\gamma-1)\frac{\lambda}{\rho}\right)\right],\\ \dot{\pi}_t &= -\lambda\left[\pi_t - \frac{(\rho+\lambda)(\gamma-1)\lambda}{\rho\gamma+(\gamma-1)\lambda}\right].\end{aligned}$$

These equations are decoupled linear differential equations with steady-state values

$$\begin{aligned}\pi_{ss} &= \frac{(\rho+\lambda)(\gamma-1)\lambda}{\rho\gamma+(\gamma-1)\lambda},\\ d_{ss} &= -\frac{(\gamma-1)(\rho+\lambda)}{\rho\gamma+(\gamma-1)\lambda} + \log\left(\gamma+(\gamma-1)\frac{\lambda}{\rho}\right),\end{aligned}$$

and the following solutions:

$$\begin{aligned}d_t - d_{ss} &= (d_0 - d_{ss})e^{-\lambda t},\\ \pi_t - \pi_{ss} &= (\pi_0 - \pi_{ss})e^{-\lambda t},\end{aligned}$$

where π_0 is given by

$$\pi_0 = \lambda(\bar{x} - d_0) = \lambda\log\left(\gamma+(\gamma-1)\frac{\lambda}{\rho}\right) - \lambda d_0.$$

Thus, we obtain the following exact solution for the non-linear system under no commitment in the limit setting with $\sigma \to 1$:

$$\begin{aligned}\log D_t &= \log D_{ss} - \log\left(\frac{D_{ss}}{D_0}\right)e^{-\lambda t},\\ \pi_t &= \pi_{ss} + \lambda\log\left(\frac{D_{ss}}{D_0}\right)e^{-\lambda t}.\end{aligned}$$

This solution implies that the saddle path has a negative slope and has the following exact form for all possible values of D:

$$\pi(D) - \pi_{ss} = -\lambda\left(\log D - \log D_{ss}\right).$$

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