Selection in Information Acquisition and Monetary Non-Neutrality*

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Abstract

Evidence on firms' expectations shows that while firms are on average uninformed about their economic environment, there is a significant amount of heterogeneity in the accuracy of their forecasts about aggregate variables and their subjective uncertainty about their own desired price changes. The natural question that follows is whose expectations matter for macroeconomic outcomes? Using data on firms' expectations, we find there is selection in information acquisition: firms that have changed their price more recently tend to have more accurate forecasts and more certain posteriors. Comparing two models with different types of information acquisition costs, we find this evidence consistent with state-dependent information acquisition where firms only acquire information when making decisions and abstain from it otherwise. Deriving a sufficient statistic for monetary non-neutrality under state-dependent information acquisition, we find that only the most informed firms' subjective uncertainty matter for the response of output to monetary shocks.

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Keywords: state-dependent information acquisition, subjective uncertainty, monetary non-neutrality

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1 Introduction

Recent evidence shows that firms are, on average, highly uninformed about their economic environment. However, there is a tremendous amount of heterogeneity in firms' uncertainty about aggregate variables, with certain firms' expectations being relatively precise and accurate while others being highly uncertain and inaccurate. This evidence raises the question that whose expectations matter for macroeconomic outcomes?

Using evidence for firms' expectations from New Zealand, we find there is selection in information acquisition: firms who have changed their prices more recently tend to have more accurate expectations about aggregate variables and are more certain about their own desired price changes. This evidence points towards a selection mechanism, according to which firms tend to acquire more information once an opportunity for a price change arrives. Motivated by this evidence, we build a theory of information choice under infrequent adjustment of prices and consider the consequences of two extreme types of cost functions for information acquisition, one linear in Shannon's mutual information and the other extremely convex.

In a model where the cost is linear in Shannon's mutual information function, we find that firms only acquire information when they change their prices and abstain from information acquisition otherwise. Accordingly, this model delivers a positive and strong relationship between firms' subjective uncertainty about their desired price and the time since its last price change. Furthermore, this model also generates a rich degree of heterogeneity in firms' subjective uncertainty that matches the shape of this variable's empirical distribution.

In contrast, in a model with an extremely convex cost for information acquisition, firms smooth their information acquisition and acquire information at a constant rate, independent of their state. Therefore, such a model fails to explain the relationship between a firm's uncertainty and the time since their last price change. Moreover, it also fails to capture the large degree of heterogeneity observed in firms' subjective uncertainty as an endogenous outcome.

As we find the evidence to favor the model with the linear cost of information acquisition, we then study the implications of this model for firms' pricing decisions and monetary non-neutrality. We find that selection in information acquisition implies that the average uncertainty among firms leads to an overestimation of monetary non-neutrality. Deriving a sufficient statistic for this non-neutrality, we find that only the price-setters' expectations matter for this macroeconomic outcome. Formally, we show that the cumulative impulse response (CIR) of output to an unexpected and permanent 1 percent decline in the aggregate price level is the sum of two terms:

$$\underbrace{\frac{1}{\theta}}_{\text{inverse frequency of price change}} + \underbrace{\frac{Z^*}{\sigma^2}}_{\text{subjective (normalized) uncertainty of price-setters}}$$

where the first term is the usual and familiar effect of price-stickiness (Alvarez, Le Bihan and Lippi, 2016). What is novel here is the second term, which captures the effect of imperfect information, and shows that only the price-setters' uncertainty affects the response of output. Since price-setters have the lowest uncertainty among all firms, this implies that average uncertainty across firms is an overestimate for the effect of imperfect information on the response of output.

Related Literature. This paper builds on and contributes several strands of literature. First, it relates to a recent literature studying how firms form their expectations and how their expectation affects their decisions. Using the survey of New Zealand firms' macroeconomic belief that we also use in this paper, Kumar, Afrouzi, Coibion and Gorodnichenko (2015) and Coibion, Gorodnichenko and Kumar (2018a) study determinants of firms' inattentiveness to aggregate economic conditions, how firms update their beliefs in a response to new information, and how changes in their belief affect their decisions. Afrouzi (2019) shows that firms facing more competitors are better informed about aggregate inflation while Yang (2020) shows firms with a greater product scope have better information about aggregate economic conditions. We use the same New Zealand survey data to motivate our new state-dependent information acquisition model.

Furthermore, our model also relates to the literature that studies the implications of different specifications for the cost of information acquisition in different settings (e.g., Dean, 2013; Hébert and Woodford, 2018; Caplin, Dean and Leahy, 2017). Our contribution to this literature is to provide evidence for the non-convexity of the cost of information acquisition and to study its macroeconomic implications.

We also contribute to the literature studying the real effects of monetary policy shocks under price stickiness or rational inattention frictions. The seminal work by Golosov and Lucas (2007) shows that a reasonably calibrated standard menu cost model cannot generate sizable monetary non-neutrality because of strong selection effects of price changes.[1]

[1] Gagnon, López-Salido and Vincent (2013) study the effect of large inflationary shocks on the timing of

Midrigan (2011) and Alvarez and Lippi (2014) introduce multi-product firms in the standard menu and find that the real effects of monetary shocks increase in the number of products firms produce. Following the seminal work by Sims (2003), the rational inattention literature provides another mechanism through which monetary policy shocks can have real effects.[2] Maćkowiak and Wiederholt (2009) develops a stylized rational inattention model and find that firms pay less attention to aggregate shocks which are less volatile than idiosyncratic shocks, leading to large monetary non-neutrality. In our model, we study both sticky prices and rational inattention in a unified framework to study the real effects of monetary policy shocks.

The theoretical model we study in Section 3 is different from previous models with both nominal rigidities and informational frictions. For example, Gorodnichenko (2008) studies a menu cost model with a partial information acquisition with a fixed observational cost. Alvarez, Lippi and Paciello (2011), Alvarez, Lippi and Paciello (2017), and Bonomo, Carvalho, Garcia and Malta (2019) study models with both menu costs and observational costs, where firms decide when they observe either idiosyncratic shocks or aggregate shocks by paying a fixed cost. In these models, firms can perfectly observe the underlying shocks that whenever they pay the fixed cost. Woodford (2009) and Stevens (2019) develop models with consideration costs, where firms' price reviews incur a fixed cost and the review decision is made on the basis of incomplete information. However, in these models, firms have perfect information once they pay the consideration costs.[3] Yang (2020) develops a model with both menu costs and rational inattention for multi-product firms and shows that price adjusters choose to be better informed about underlying shocks. This selection effect in information processing leads to a leptokurtic distribution of firms' desired price changes.[4] Our new contribution to this literature is to develop a continuous-time model with both rational inattention and nominal rigidities and study its implications for monetary non-neutrality in an analytical framework.

price changes using Mexican CPI data and find direct support for a selection effect. Carvalho and Kryvtsov (2018) finds evidence of strong price selection across goods and services using detailed micro-level consumer price data for the UK, the US, and Canada.

[2] See, for instance, Sims (2010) and Maćkowiak, Matějka and Wiederholt (2018) for comprehensive reviews.

[3] Models with menu costs and exogenous information rigidities include Klenow and Willis (2007), Knotek (2010), Hellwig and Venkateswaran (2015), and Baley and Blanco (2019) among others.

[4] In menu costs literature, many previous studies assume a fat-tailed distribution of idiosyncratic shocks to fit the distribution of micro price data. See, for instance, Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019.

2 Motivating Evidence

In this section, we provide our motivating evidences on firms' belief formation using a quantitative survey of firms' expectations in New Zealand.[5] This survey was conducted in multiple waves among a random sample of firms in New Zealand with broad sectoral coverage.[6] We provide three empirical results that motivate our new state-dependent rational inattention model in Section 3. First, there is a lot of heterogeneity in firm-level subjective uncertainty. Second, the distributions of firms' desired price changes and revisions in their price gaps are both leptokurtic. Third, there is a positive relationship between time since last price change and the accuracy of firms' inflation expectations as well as their subjective uncertainty about their desired price changes.

2.1 Heterogeneity in Firms' Subjective Uncertainty

We first show that there is a significant degree of heterogeneity in firms' subjective uncertainty. The New Zealand survey data allows us to measure firms' subjective uncertainty about their desired price changes. We define a firm's desired price change as the amount by which the firm would change its price if it could freely do so. Firms in the survey were asked to assign probabilities (from 0 to 100) to the different values for their current desired price changes. We calculate the standard deviation—which is a measure of firms' subjective uncertainty—surrounding firms' desired price changes using the implied probability distribution. Figure 1 shows the distributions of firms' subjective uncertainty in different sectors. First, subjective uncertainty is highly dispersed across firms. Second, this heterogeneity exists both within and across sectors.

2.2 Price Changes and Subjective Uncertainty

Our last empirical result is on the relationship between firms' subjective uncertainty and their recent price changes. We investigate if firms that adjusted their prices recently differ

[5] See Coibion et al. (2018a) and Kumar et al. (2015) for a comprehensive description of the survey.

[6] Several papers use the data to characterize how firms form their expectations. For example, Afrouzi (2019) shows that strategic complementarity decreases with competition, and documents that firms with more competitors have more certain posteriors about the aggregate inflation. Also, Coibion, Gorodnichenko, Kumar and Ryngaert (2018b) evaluate the relation between first-order and higher-order expectations of firms, including how they adjust their beliefs in response to a variety of information treatments. Yang (2020) shows that firms producing more goods have both better information about inflation and more frequent but smaller price changes.

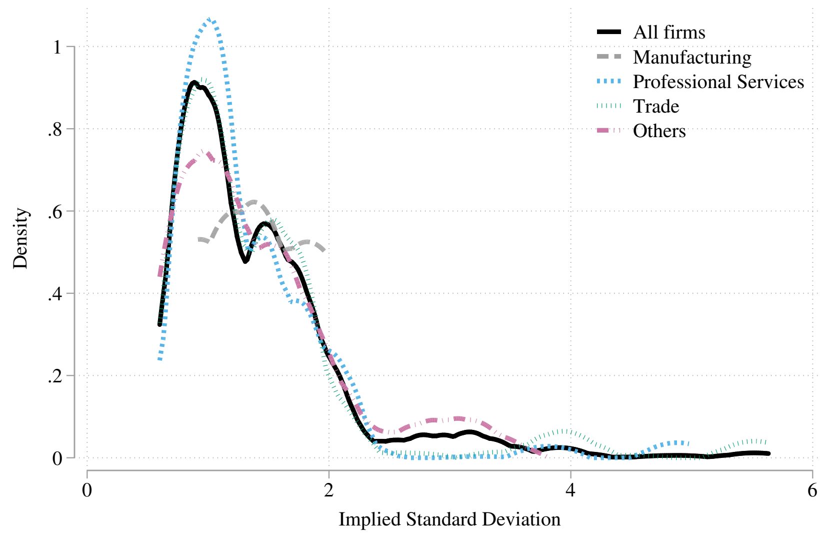

Figure 1: Subjective Uncertainty about Firms' Desired Price Changes

Notes: This figure plots distributions of firms' subjective uncertainty about their desired price changes in different sectors in #9 wave of the survey data. The subjective uncertainty is measured by the standard deviation implied by the reported probability distribution over different outcomes of firms' desired price changes if firms are free to change their prices.

from others in their subjective uncertainty about their desired price changes. Column (1) of Table 1 shows that controlling for industry fixed effects, firms that have adjusted their prices within the past 12 months have smaller standard deviation of the distribution of their desired price changes. This negative relationship holds if we control for firm-level characteristics (Column (2)) and manager characteristics (Column (3)).[7] Moreover, Column (4) shows that firms with longer elapsed time since price change have greater subjective uncertainty about their desired price changes. This suggests that there is a negative correlation between firms' subjective uncertainty and having a recent price change.

A high degree of uncertainty about the desired price change does not necessarily mean that the firm is less informed about it. Since we cannot observe firms' true optimal price in data, we cannot directly calculate a gap between firms' perceived optimal price and their true optimal price. Instead, we use firms' nowcast errors about aggregate inflation to investigate the relationship between firms' attentiveness and their recent price changes. We

[7]Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, the slope of the profit function, firms' expected size of price changes in 3 months, and firms subjective uncertainty about their desired prices in # 8 wave. Manager controls include the age of the respondent (each firm's manger), education, and tenure at the firm.

Table 1: Recent Price Changes and Subjective Uncertainty

	(1)	(2)	(3)	(4)
Dependent variable: Subjective uncertainty about firms' desired price changes				
Dummy for price changes (last 12 months)	-0.112* (0.057)	-0.210*** (0.063)	-0.265*** (0.056)	
Time elapsed since price change				0.010* (0.005)
Observations	485	488	486	487
R-squared	0.061	0.170	0.243	0.188
Industry fixed effects	Yes	Yes	Yes	Yes
Firm-level controls		Yes	Yes	Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the subjective uncertainty about firms' desired price changes over the next three months from #9 wave of the survey. The subjective uncertainty is measured by the standard deviation implied by the reported probability distribution over different outcomes of firms' desired price changes if firms are free to change their prices. Industry fixed effects include dummies for 13 sub-industries. Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, the slope of the profit function, firms' expected size of price changes in 3 months, and firms subjective uncertainty about their desired prices in # 8 wave. Manager controls include the age of the respondent (each firm's manger), education, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit Australian and New Zealand Standard Industrial Classification (ANZ SIC) level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

define firms' inflation nowcast errors as their absolute errors in regard to inflation rates over the last 12 months. Then, we investigate if firms that have adjusted their prices recently differ from others in their errors. Column (1) of Table 2 shows that controlling for industry fixed effects, firms that adjusted their prices recently have smaller inflation nowcast errors. As suggested in Column (2), one might think that the nowcast errors are larger for price non-adjusters since they are more likely to have longer duration of price reviews. In Columns (3) and (4), we show firms that changed their prices recently have smaller inflation nowcast errors even controlling for firms' frequency of price reviews and other firm-level and manager controls. This suggests that there is a positive correlation between being informed and having a recent price change.

Table 2: Recent Price Changes and Attentiveness to Aggregate Inflation

	(1)	(2)	(3)	(4)
Dependent variable: Inflation nowcast errors				
Dummy for price changes	-0.226***		-0.171***	-0.173*
(last 3 months)	(0.061)		(0.061)	(0.095)
Months between price reviews		0.023***	-0.006	0.001
		(0.007)	(0.006)	(0.008)
Observations	2,874	2,846	2,889	1,348
R-squared	0.824	0.838	0.820	0.835
Industry fixed effects	Yes	Yes	Yes	Yes
Firm-level controls			Yes	Yes
Manager controls				Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the absolute value of firm errors about past 12 month inflation from #1 wave of the survey. Industry fixed effects include dummies for 17 sub-industries. Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit Australian and New Zealand Standard Industrial Classification (ANZ SIC) level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

3 Model

3.1 Environment

Time is continuous and there is a unit measure of firms in the economy, indexed by i.

Shocks and Payoffs. Each firm i tracks an *ideal price* $p^*_{i,t}$. We assume that this ideal price is exogenous to the problem of the firm and is a Brownian motion with drift μ and scale σ with increments that are independent across firms:

$$dp^*_{i,t} = \mu dt + \sigma dW_{i,t}.$$

Firms are price setters and subject to a Calvo friction. Formally, the opportunity of changing the price is exogenous, independent across firms and governed by a Poisson process with arrival rate θ. Therefore, the time until the next price change for any firm is exponentially distributed with the same rate, and θ constitutes the average frequency of price changes in this economy.

Moreover, the instantaneous payoff of firm i at time t is given by

$$-B(p_{i,t} - p^*_{i,t})^2,$$

where the quadratic term captures the firm's loss from mis-pricing their product, with B measuring of the concavity of the firm's profit function.

Information Structure and Cost of Attention. We assume firms cannot directly observe their ideal prices but can acquire information about it subject to a cost. Formally, firm i observes a signal process $\{s_{i,t} : t \geq 0\}$ over time that is informative about $p^*_{i,t}$:

$$ds_{i,t} = p^*_{i,t} dt + \sigma_{s,i,t} dW_{s,i,t}, \tag{3.1}$$

where $W_{s,i,t}$ is a standard Brownian motion, independent of $p^*_{i,t}$, that captures the measurement error of firm i of $p^*_{i,t}$.

Firms are inattentive in the sense that they choose the precision of these signals over time through picking a sequence $\{\sigma_{s,i,t} \in \mathbb{R}_+ \cup \{\infty\}, t \geq 0\}$, where we define $\sigma_{s,i,t} \equiv \infty$ as an instance in which $ds_{i,t} = 0$. We assume that at any given point in time, firms have form their beliefs using the set of all their previous signals, denoted by $S_{i,t} \equiv \{s_{i,\tau} : \tau \leq t\}$. Moreover, we assume that the cost of information for firm i in picking the precision of their signals is given by $C(\mathbb{I}(p^*_{i,t}|S_{i,t}))$, where $C(.)$ is an increasing and weakly convex function, and $\mathbb{I}(s_{i,t}, p^*_{i,t})$ is define the reduction rate in entropy of $p^*_{i,t}$ at time t:

$$\mathbb{I}(p^*_{i,t}|S_{i,t}) \equiv \lim_{dt \downarrow 0} \frac{h(p^*_{i,t}|S_{i,t-dt}) - h(p^*_{i,t}|S_{i,t})}{dt},$$

where $h(.)$ is the differential entropy function.

Regarding the function $C(.)$, we focus on two limiting specifications in our analysis in terms of convexity, which also capture two common cases in the rational inattention literature:

Linear. Our first specification is to assume that the cost is non-convex:

$$C(x) = \underline{c}(x) \equiv \omega x \tag{3.2}$$

This functional form coincides with one side of the classic rational inattention models, in which the cost is linear in Shannon's mutual information function.

Extremely Convex (Fixed Capacity). Our second specification is to assume an extremely convex functional form for the cost of attention:

$$C(x) = \bar{c}(x) \equiv \begin{cases} 0 & x \leq \bar{\lambda} \\ \infty & x > \bar{\lambda} \end{cases} \tag{3.3}$$

This functional form captures the other common case in the rational inattention literature which assumes that agents only have a fixed capacity for processing information, which they cannot exceed.

Firms' Problems. Firm i's problem is to choose a set of precisions for their information set, along with a set of planned prices that are implemented upon the arrival of an opportunity for a price change:

$$\begin{aligned}
&\ell_{i,0} \equiv \\
&\min_{\{\sigma_{s,i,t}\geq 0, \tilde{p}_{i,t}: S_{i,t}\to\mathbb{R}\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}[\underbrace{B(p_{i,t}-p^*_{i,t})^2}_{\text{loss from mis-pricing}} + \underbrace{C(\mathbb{I}(p^*_{i,t}|S_{i,t}))}_{\text{cost of information}}]dt \middle| S_{i,0}\right] && s.t. \\
&dp_{i,t} = (\tilde{p}_{i,t} - p_{i,t})d\chi_{i,t}, && \forall t \geq 0 \\
&S_{i,t} = \{s_{i,\tau} : 0 \leq \tau \leq t\} \cup S_{i,0}, && \forall t \geq 0 \\
&ds_{i,t} = p^*_{i,t}dt + \sigma_{s,i,t}dW_{s,i,t}, && \forall t \geq 0 \\
&S_{i,0}, p_{i,0} \text{ given.}
\end{aligned}$$

where $\chi_{i,t}$ is the Poisson process governing the arrival of price changes, $\tilde{p}_{i,t}$ is the firms' *planned price* for time t, that is implemented as the firm's *actual price* (denoted by $p_{i,t}$) if a Calvo shock arrives ($d\chi_{i,t} = 1$).

3.2 Characterization

In this section we characterize the solution to firms' problem for the two types of the cost function specified in Equation (3.2) and Equation (3.3).

Evolution of Beliefs. We start by characterizing the evolution of firms' beliefs for an arbitrary information acquisition strategy over time. Second, we solve the problem by deriving the implied Hamilton-Jacobi-Bellman equation.

Lemma 3.1 (***Evolution of Beliefs***)**.** *Given a sequence of precisions* $\{\sigma_{s,i,t} \geq 0 : t \geq 0\}$*, and an initial Guassian information set* $S_{i,0}$*, let* $S_{i,t} = \{s_{i,\tau} : 0 \leq \tau \leq t\} \cup S_{i,0}$ *denote a firm's information set at time* t *where* $s_{i,t}$ *evolves according to Equation* (3.1)*. Then,*

1. *the firm's belief about* $p^*_{i,t}$ *conditional on* $S_{i,t}$ *is given by*

$$\begin{aligned} & p^*_{i,t}|S_{i,t} \sim \mathcal{N}(\hat{p}_{i,t}, z_{i,t}), && (3.4) \\ & d\hat{p}_{i,t} = \lambda_{i,t}(p^*_{i,t} - \hat{p}_{i,t})dt + \sqrt{\lambda_{i,t} z_{i,t}}dW_{s,i,t} && \textit{(evolution of the mean)} \\ & dz_{i,t} = (\sigma^2 - \lambda_{i,t} z_{i,t})dt && \textit{(evolution of the variance)} \\ & \hat{p}_{i,0}, z_{i,0} \textit{ given.} \end{aligned}$$

   where $\lambda_{i,t} \equiv z_{i,t}/\sigma^2_{s,i,t}$ *is the Kalman-Bucy gain of* i *at* t*.*

2. *the rate of reduction in entropy at time* t *for firm* i *is the Kalman-Bucy gain:*

$$\mathbb{I}(p^*_{i,t}|S_{i,t}) = \lambda_{i,t}.$$

The first part of the lemma takes advantage of the normality assumption and characterizes the evolution of these beliefs for a given choice of precisions. The second part of the lemma derives a representation for the cost of information in this setup. The linearity of this cost in the Kalman-Bucy gain gives an intuitive interpretation to the information acquisition problem of the firm: a higher signal precision at time t implies a higher Kalman-Bucy gain which implies a higher cost of information acquisition. Moreover, given that $z_{i,t}$ is predetermined at time t by the past choices of the firm and its initial information structure, a choice for $\sigma_{s,i,t} \geq 0$ maps one to one to a choice of a Kalman-Bucy gain, as $\lambda_{i,t} = z_{i,t}/\sigma^2_{s,i,t} \geq 0$.

to layout an intuitive framework for the discussion of our results, it is useful to define three wedges that create the appropriate terminology for interpreting the implications of each friction for the behavior of the firm.

Definition 3.1. *We define firm* i*'s* ***true price gap****,* ***perceived price gap****, and* ***belief gap*** *at* *time t as*

$$\begin{aligned} x^*_{i,t} &\equiv p^*_{i,t} - p_{i,t}, \\ x_{i,t} &\equiv \mathbb{E}[p^*_{i,t}|S_{i,t}] - p_{i,t}, \\ b_{i,t} &\equiv p^*_{i,t} - \mathbb{E}[p^*_{i,t}|S_{i,t}], \end{aligned}$$

respectively.

The true price gap is the payoff relevant statistic for a firm that shows up in their instantaneous payoff and a firm always prefers a smaller true price gap. The perceived price gap and the belief gap are then a decomposition of this true price gap:

$$\underbrace{x^*_{i,t}}_{\text{true price gap}} = \underbrace{x_{i,t}}_{\text{perceived price gap}} + \underbrace{b_{i,t}}_{\text{belief gap}}$$

This decomposition is useful because it separates the role of each of the model's two frictions in the firms' payoff. Holding beliefs fixed, the perceived price gap, $x_{i,t}$, captures how much the firm's price is away from *what they believe* is to be their ideal price. The belief gap, however, disregards the nominal rigidity and captures how far that belief is from the truth.

It follows from Lemma (3.1) that conditional on a firm's information set at a given time t, their belief gap at that time is normally distributed according to

$$b_{i,t}|S_{i,t} \sim \mathcal{N}(0, z_{i,t}),$$

where $z_{i,t}$ is their uncertainty about their ideal price as in Equation (3.4).

Lemma 3.2. *A firm's **perceived instantaneous loss** from mis-pricing can be decomposed as*

$$\underbrace{\mathbb{E}[{x^*_{i,t}}^2|S_{i,t}]}_{\text{perceived loss}} = \underbrace{x^2_{i,t}}_{\text{perceived price gap}^2} + \underbrace{z_{i,t}}_{\text{subjective uncertainty}}$$

Moreover, at any time t, the problem of a firm with perceived price gap x and subjective *uncertainty z is characterized by the following HJB equation:*

$$
\begin{aligned}
\rho\ell(x,z) = B(x^2+z) + \sigma^2\partial_z\ell(x,z) + \mu\partial_x\ell(x,z) + \theta[\ell(\tilde{x},z) - \ell(x,z)] \\
+ \min_{\lambda\geq 0}\left\{[\frac{1}{2}\partial_{xx}\ell(x,z) - \partial_z\ell(x,z)]\lambda z + C(\lambda)\right\}, \\
\tilde{x} \equiv \arg\min_x \ell(x,z) = -\frac{\mu}{\rho+\theta}
\end{aligned}
$$

The Lemma shows that a firm's perceived price gap and subjective uncertainty along with the state of their Poisson shock are sufficient state variables for characterizing their dynamic problem at any moment in time. Moreover, $\tilde{x} = -\frac{\mu}{\rho+\theta}$ is the reset perceived price gap for when firms get to reset their price. It is a negative quantity because firms expect to be stuck with their price for some time while their ideal price grows with drift μ.

We are now ready to state our main results.

Theorem 3.1. ***(Optimal Information Acquisition with Linear Cost)*** *Suppose the cost of information acquisition is linear is Shannon's mutual information function (Specification 3.2). Then,*

1. *It is optimal for a firm to never acquire information in between price changes.*
2. *Upon the arrival of an opportunity for a price change for a firm with uncertainty z, there exists a baseline uncertainty $Z^* > 0$ such that,*
   (a) if $z \leq Z^*$, the firm acquires no information;
   (b) if $z > Z^*$, the firm acquires enough information to reset its uncertainty to Z^*,

   where Z^* solves:

$$
\frac{1}{Z^*} = \frac{B}{\omega(\rho+\theta)} + \theta\int_0^\infty e^{-(\rho+\theta)h}\frac{1}{Z^*+\sigma^2 h}dh \tag{3.5}
$$

The main take away from Theorem 3.1 is that there is *selection* in information acquisition in the sense that only firms who are changing their prices acquire information. Furthermore, once a firm does acquire information, they do so much to drive down their uncertainty about their ideal price to a baseline level. The costly nature of attention in this model implies that this baseline uncertainty is not zero; meaning that while the uncertainty about the ideal price is at its lowest among price changers, it is still positive as in Equation (3.5). It is straight forward to see that this uncertainty is decreasing with the concavity of the profit function, and increasing with the cost of attention, the variance of the innovations to the ideal price and the discount factor of the firm. In fact, for $\rho \to 0$ and $\theta \to 0$, we can simplify its expression with the following approximation:

$$\lim_{\substack{\rho\to 0\\ \theta\to\infty}} Z^* = \sigma\sqrt{\omega B^{-1}}.$$

Proposition 3.1. ***(Optimal Information Acquisition with Extremely Convex Cost)*** *Suppose the cost of information acquisition is extremely convex in Shannon's mutual information function (Specification 3.3). Then, all firms acquire information at a constant Kalman-Bucy gain of $\bar{\lambda}$, independent of their state. In particular, given an initial belief $x_0^*|S_0 \sim N(x_0, z_0)$, the firm's uncertainty evolves according to*

$$z_t = z_0 e^{-\bar{\lambda}t} + \frac{\sigma^2}{\bar{\lambda}}(1 - e^{-\bar{\lambda}t})$$

which converges to the stationary variance of $\frac{\sigma^2}{\bar{\lambda}}$ as $t \to \infty$.

4 Model Predictions and Relation to Evidence

So far, we have shown that the degree of convexity in the cost of information acquisition has significant implications for firms' information acquisition strategy. In this section, we compare the predictions of these models to our motivating evidence.

4.1 Uncertainty and Time Since Last Price Change

One the most salient differences between the information acquisition strategies under the models is their implications for the relationship between firms' subjective uncertainty and time since their last price change.

In the model with the linear cost of information acquisition, since firms do not acquire information in between price changes, there is a linear relationship between the time since a firm's last price change and its subjective uncertainty. In particular, the uncertainty of a firm that changed their price h periods ago is simply given by

$$Z_h = Z^* + \sigma^2 h$$

which is consistent with our findings in Table 1.

Alternatively, in the model with the extremely convex cost for information acquisition, uncertainty of firms is independent of the time since their last price changes, as shown in Proposition 3.1.

4.2 Cross-sectional Distribution of Subjective Uncertainty

In the model with the linear cost of information acquisition, since firms update their information infrequently, the model implies cross-sectional heterogeneity in uncertainty of firms in the equilibrium. This is a feature that only emerges from the combination of the two frictions of the model and would fade away in absence of either one of them: in a model with no nominal rigidities, all firms update their information all the time, and all firms have the same uncertainty about their ideal price; similarly, in a model with only nominal rigidities but full information, all firms' uncertainty is trivially zero.

Alternatively, in the model with the extremely convex cost of information acquisition, all firms acquire information at the same rate and consequently have the same uncertainty in the limit.

Proposition 4.1. *The time-invariant distribution of firms' subjective uncertainty about their ideal prices,*

1. *in the model with linear cost of information acquisitions, is an exponential distribution with rate $\frac{\theta}{\sigma^2}$, shifted by Z^*. Formally, letting $N_Z(z)$ denote the CDF of this distribution. Then,*

$$N_Z(z) = \begin{cases} 0 & z < Z^* \\ 1 - e^{-\frac{\theta}{\sigma^2}(z - Z^*)} & z \geq Z^* \end{cases}$$

2. *in the model with the extremely convex cost of information acquisition is an mass-point at $\frac{\sigma^2}{\lambda}$.*

This Proposition shows that, with linear cost of information, uncertainty across firms inherits the exponential distribution of time between price changes. As firms change their prices, they reset their uncertainty to Z^*, after which their uncertainty grows linearly in time, with slope σ^2, until the next opportunity for a price change arrives. This is consistent with Figure 1, which depicts a large degree of heterogeneity in firms' subjective uncertainty.

Therefore, the evidence favors the model with the linear cost of information acquisition, but are not consistent with the predictions of the model with the convex cost, in which average uncertainty is the same as uncertainty conditional on a price change. In fact, a main take-away from the last Proposition under the linear cost specification, which is consistent with the evidence, is that the average uncertainty across firms is higher than the uncertainty conditional on a price change.

Corollary 4.1. *Let $\bar{Z}$ denote the unconditional mean of uncertainty across firms. Then,*

$$\bar{Z} = Z^* + \frac{\sigma^2}{\theta}$$

where Z^* is the average uncertainty among firms when they change their prices.

4.3 Distributions of Price Changes and Price Gaps

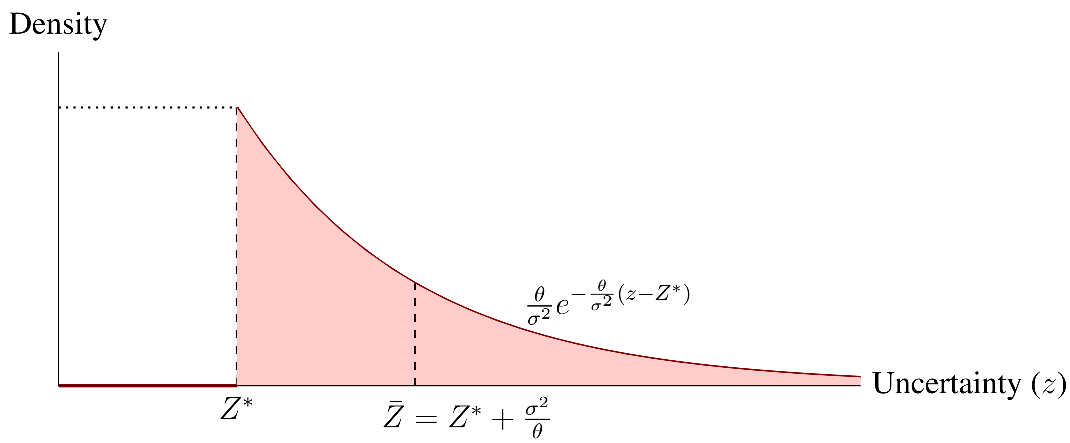

Figure 2: Distribution of Uncertainty Across Firms

Proposition 4.2. *The time invariant distribution of price changes is an asymmetric Laplace distribution with location 0, scale $\frac{\sqrt{2\theta}}{\sigma}$ and asymmetry $\sqrt{1+\frac{\mu^2}{2\theta\sigma^2}} - \sqrt{\frac{\mu^2}{2\theta\sigma^2}}$.*

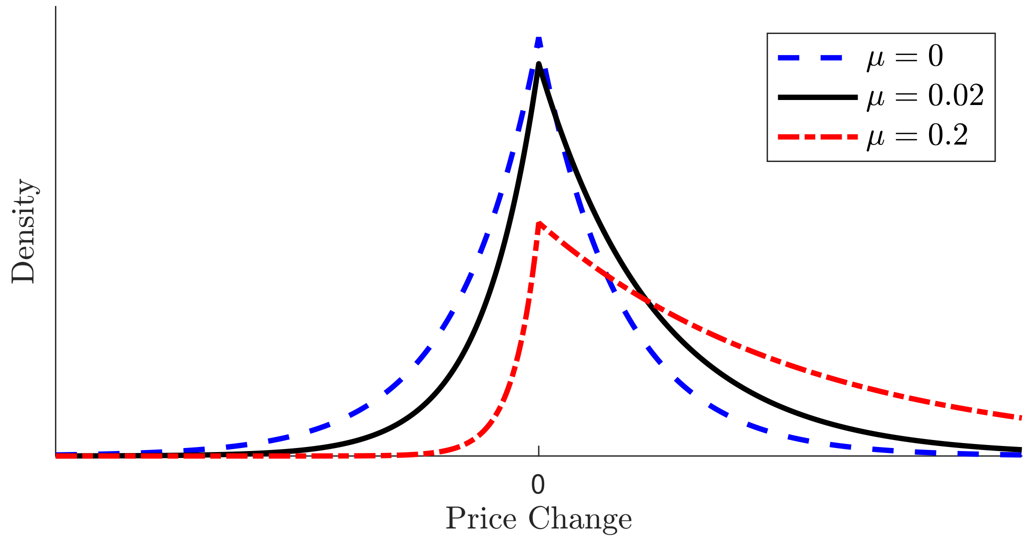

Figure 3: Distribution of Price Changes Across Firms

Corollary 4.2. *Inattention cannot be identified from distribution of price changes. Formally, the distribution of price changes is invariant with respect to degree of inattention.*

Proposition 4.3. *The time-invariant distribution of true price gaps across firms, x^*, has a normal-Laplace distribution; it can be decomposed as*

$$x^* = x_n + x_w$$

where $x_n \sim \mathcal{N}(0, Z^*)$ and $x_w \sim A - Laplace(\frac{\mu}{\rho+\theta}, \frac{\sqrt{2\theta}}{\sigma}, \sqrt{1 + \frac{\mu^2}{2\theta\sigma^2}} - \sqrt{\frac{\mu^2}{2\theta\sigma^2}})$ are independent random variables.[8]

Normal-Laplace distributions inherit the properties of the their normal component in the middle but the tail behavior of their Laplace component. Figure (4) shows the implied normal-Laplace distribution for the case of $\mu = 0$.

[8]For a detailed discussion of normal-Laplace distributions see [Cite Reed (2004)].

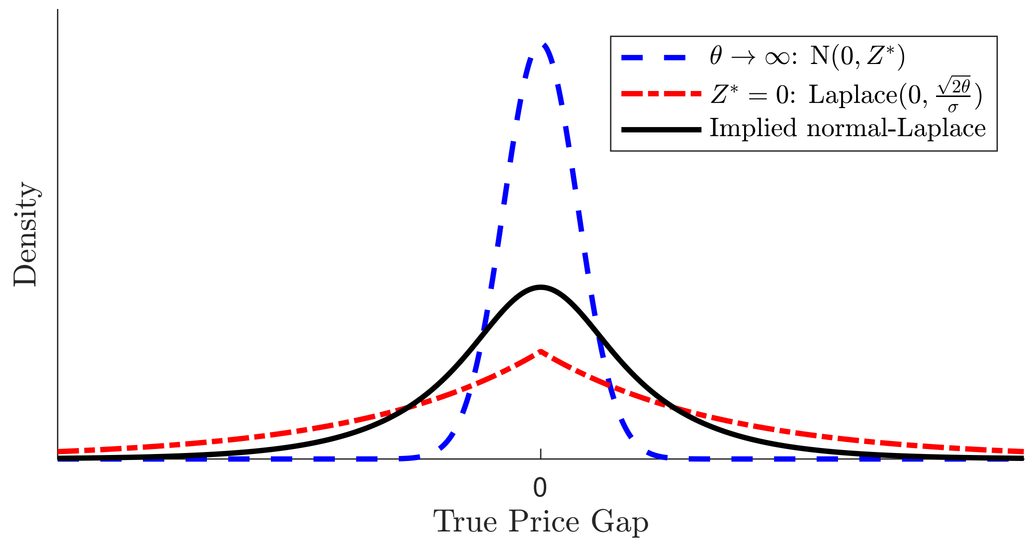

Figure 4: Distribution of True Price Gaps Across Firms

5 Implications for Monetary Non-Neutrality

In this section, we investigate the implications of our model for monetary non-neutrality. To this end, we assume that the output gap of a firm is proportional to their true price gap – a benchmark result in monetary models:[9]

$$y_{i,t} \equiv \epsilon x^*_{i,t} = \epsilon(b_{i,t} + x_{i,t}),$$

where ϵ can be interpreted as the inter-temporal elasticity of substitution for a representative household, and the second part directly follows from Definition (3.1).

To avoid the complications of thinking about positive versus negative monetary shocks, for the remainder of this section we will assume $\mu = 0$. However, later on, in characterizing the time-invariant distribution of prices and price changes, we will allow for non-zero drift.

A convenient implication of $\mu = 0$ is that that the distribution of perceived price gaps collapses to a degenerate distribution at zero, meaning that $x_{i,t} = 0$ for all i and t. Thus,

$$y_{i,t} = \epsilon b_{i,t}$$

which means that in order to understand the behavior of output gaps under the optimal

[9] See [Cite Alvarez] for instance.

information acquisition policy of firms, we need to specify how belief gaps evolve. Since firms update their information only when they change their prices, belief gaps inherit the Poisson process of price changes and firms end up revising their beliefs either not by much or by a lot. Since output gaps are proportional to belief gaps, they behave exactly the same way. The following Lemma characterizes this result.

Lemma 5.1. *The output gap of a firm is a Brownian motion with a Poisson jump given by:*

$$\begin{aligned} db_{i,t} &= \sigma dW_{i,t} - [\lambda_{i,t} b_{i,t} + U_{i,t}] d\chi_{i,t} \\ dz_{i,t} &= \sigma^2 dt + [Z^* - z_{i,t}] d\chi_{i,t} \\ \lambda_{i,t} &= 1 - \frac{Z^*}{z_{i,t}},\, U_{i,t} \sim \mathcal{N}(0, \lambda_{i,t} Z^*) \end{aligned}$$

where $\sigma dW_{i,t}$ is the innovation to the firm's ideal price, $\chi_{i,t}$ is the Poisson r.v. governing the arrival of a price change, and $U_{i,t}$ is the firm's mistake in observing the ideal price.

The Lemma also shows that how the uncertainty of a firm at the time of a price change affects the size of their belief revision. To see this, recall that $\lambda_{i,t}$ was the "amount" of information that a firm acquires which depends on the prior uncertainty of firms upon updating its information and ranges from 0, when $z_{i,t} = Z^*$, to 1, when $z_{i,t} \to \infty$. Importantly, the size of the Poisson jump in the belief gap is determined by $\lambda_{i,t}$ which introduces a new source of selection: firms who have not changed their prices for a longer time, acquire more information upon the arrival of an opportunity for a price change, and their realized belief gaps are smaller on average.

Notice that this selection would be absent if either friction was eliminated. In absence of information rigidities when $Z^* = 0$, all firms have $\lambda = 1$ and they all fully revise their beliefs so that their belief gap is reset to zero. In that sense, the limit where $Z^* = 0$ collapses to a model where firms acquire full information upon updating their information.

Having specified the evolution of these gaps, we can now characterize the degree of monetary non-neutrality. To do so, we start by defining monetary non-neutrality for one firm, and then we provide an aggregation result.

5.1 Output Gap of Individual Firms

To shed light on how heterogeneity in uncertainty leads to differential response in production of firms, we start by characterizing the life-time production of a single firm.

Definition 5.1. *We define the expected life-time output gap of firm i at time 0 as*

$$Y_{i,0} = \mathbb{E}\left[\int_0^\infty y_{i,t}dt\right]$$

The question of monetary non-neutrality in this model is that how an initial shock to a firm's variables would persist over time. In particular we will interpret a monetary shock as a one time unanticipated shift in firms' true price gaps. Alternatively, using the definition of output gap in Equation (5), one can interpret such a shift as a shock to firms' initial output gap. The following Proposition shows that a firm's uncertainty and initial output gap are sufficient for characterizing their expected life-time output gap.

Lemma 5.2. *The expected life-time output of firm i at any given time is uniquely determined by their initial output, y, and initial uncertainty, z:*

$$Y(y,z) = m(z)y, \quad \underbrace{m(z) \equiv \frac{1-(\bar{\lambda}_z - \bar{\lambda}_{Z^*})}{\theta\bar{\lambda}_{Z^*}}}_{\text{output multiplier}},$$

where

$$\lambda_z \equiv \mathbb{E}^{z'}[1-\frac{Z^*}{z'}|z' \geq z] = 1-\frac{\theta}{\sigma^2}e^{\frac{\theta}{\sigma^2}z}\Gamma(0,\frac{\theta}{\sigma^2}z)$$

is the expected information acquisition of a firm whose uncertainty is z. Here $\Gamma(.,.)$ is the upper incomplete Gamma function.

The Lemma shows that the expected life-time production of a firm is a multiple of their initial output gap y, where the multiplier depends on the firm's initial uncertainty. Note that this dependence relies on both frictions and would go away in absence of either of them. To see this, it is useful to consider the two benchmarks:

1. **The case of** $Z^* = 0$: in this case, which is the limit of the model where the only friction is the nominal rigidity, the output multiplier is simply the inverse of the frequency of price change, $1/\theta$. The intuition is simple: any initial shock to a firm's output gap would last until their next price change, after which the firm would adjust and reset the gap to zero. Hence, the life-time expected output gap of a firm is the average time between price adjustments, θ^{-1}.

2. **The case of** $\theta \to \infty$: in this case, which is the limit of the model where the only

friction is the costly information, the output multiplier converges to $\bar{\lambda}_{\theta\to 0}^{-1} = \sigma^2/Z^*$. The intuition is that any shock to a firm's output gap would only persist because the firm does not have enough information to fully adjust their price and close the gap. Since $\bar{\lambda}$ is the rate of information acquisition in this economy, the higher $\bar{\lambda}$ the sooner the firm will be able to close their output gap. Hence, the expected life-time output gap of a firm is related to the inverse of the rate of information acquisition.

This dependence of the output multiplier on initial uncertainty creates heterogeneity on how firms respond to the same initial shock.

Proposition 5.1. *The output multiplier of a firm is decreasing in their initial uncertainty z:*

$$m'(z) = \frac{\lambda_z - \bar{\lambda}_z}{\theta \bar{\lambda}_{Z^*}} \leq 0$$

The Lemma shows that firms with higher uncertainty have smaller multipliers. Therefore, two firms with the same initial shock to the output gap would respond differently depending on their initial uncertainty. Firms with higher uncertainty on average acquire more information when they change their prices and therefore are more successful in closing their output gap.

5.2 The Aggregate Multiplier

Definition 5.2. *For an initial joint distribution of output gap and uncertainty, $N(y,z)$, we define the associated cumulative response of output as*

$$\mathcal{M}(N) \equiv \int Y(b,z) dN(y,z).$$

Therefore $\mathcal{M}(N)$ captures the cumulative response of output when output gap and uncertainty in the economy is initially distributed according to N.

Proposition 5.2. *For an initial joint distribution $N(y,z)$, the cumulative response of output associated with N is*

$$\mathcal{M}(N) = \underbrace{\mathbb{E}^N[m(z)]}_{\text{average multiplier}} \times \underbrace{\mathbb{E}^N[y]}_{\text{average initial gap}} + \underbrace{cov^N(m(z), y)}_{\text{selection effect}}$$

Proposition 5.3. *Let $\tilde{N}(y, z)$ denote the time-invariant joint distribution of output and uncertainty implied by the model. Then,*

1. *The cumulative response of output associated with $\tilde{N}$ is zero: $\mathcal{M}(\tilde{N}) = 0$.*

2. *Let $N_\delta(y, z) = \tilde{N}(y - \delta, z)$ be the initial distribution associated with a one-time unanticipated shock that increases the output gap of all firms by δ. Then,*

$$\mathcal{M}(N_\delta)/\delta = \frac{1}{\theta} + \frac{Z^*}{\sigma^2}$$

The Proposition shows that only the uncertainty of price-setters matter for monetary non-neutrality.

Corollary 5.1. *Monetary non-neutrality cannot be identified from the distribution of price changes.*

Proof follows from the fact that the distribution of price changes is invariant to the degree of inattention.'

6 Conclusion

In this paper we study the implications of state-dependent information acquisition for macroeconomic outcomes. While empirical evidence shows that firms on average are highly uninformed about aggregate variables, in a model with endogenous information acquisition, we show that only the expectations of the most informed firms matter for monetary non-neutrality and evolution of prices.

Our findings provide a new perspective on communication policies that target expectations of firms. Since we find that firms only acquire information when an opportunity for a price change arrives, our model favors targeted communications policies that recognizes these selection effects in information acquisition.

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Appendices

A Proofs

[TO BE ADDED]

Proof of Lemma (5.2). Recall that $y_{i,t} = \epsilon(b_{i,t} + x_{i,t})$. Hence, with $\mu = 0$,

$$Y_{i,0} = \epsilon\mathbb{E}[\int_0^1 b_{i,t}].$$

where the the belief gaps evolve according to Equation (5.1). Therefore, the expectation above is pinned down by the initial belief and perceived price gap. Also, due to the linear relationship between perceived price gap and time since last price change we have

$$x_{i,0} = \frac{\mu}{\sigma^2}(z_{i,0} - Z^*) - \frac{\mu}{\rho + \theta}.$$

Thus,

$$Y_{i,0} = Y(b_{i,0}, z_{i,0}).$$

For simplicity of notation let us drop the subscripts. It is straight forward to show that given the process of belief gap and uncertainty, $Y(.,.)$ should solve the following PDE:

$$\begin{aligned} Y(b,z) = \frac{\epsilon}{\theta}b + \frac{\sigma^2}{2\theta}\partial_{bb}Y(b,z) + \frac{\sigma^2}{\theta}\partial_z Y(b,z) \\ + \mathbb{E}^U[Y((1-\lambda(z))b + U, Z^*)], \end{aligned}$$

Where $\lambda(z) =$Despite the complicated nate of this PDE, it is clear from the definition of $Y_{i,0}$ and the evolution of the belief gap that $Y(b,z)$ should be linear in b. Thus, we look at solutions of type:

$$Y(b,z) = f(z)b + g(z).$$

Plugging this general solution to the PDE, we get

$$\begin{aligned} f(z)b + g(z) = [\frac{\epsilon}{\theta} + \frac{\sigma^2}{\theta}f'(z) + f(Z^*)(1-\lambda(z))]b \\ + \frac{\sigma^2}{\theta}g'(z) + g(Z^*) \end{aligned}$$

The relevant boundary conditions are $\lim_{z\to\infty} f(z) = \frac{\epsilon b}{\theta}$ and $g(Z^*) = 0$. Therefore, $g(z) = 0$ and

$$\begin{aligned} f(z) &= \frac{\epsilon}{\theta}\left(1 + \frac{1 - \mathbb{E}^{z'}[\lambda(z')|z' \geq z]}{\mathbb{E}^{z'}[\lambda(z')]}\right) \\ &= \frac{\epsilon}{\theta}\left(1 + \frac{\alpha Z^* e^{\alpha z}\Gamma(0, \alpha z)}{1 - \alpha Z^* e^{\alpha Z^*}\Gamma(0, \alpha Z^*)}\right) \end{aligned}$$

where $\alpha \equiv \frac{\theta}{\sigma^2}$. ■