Rational Inattention, Sticky Prices and Monetary Non-Neutrality

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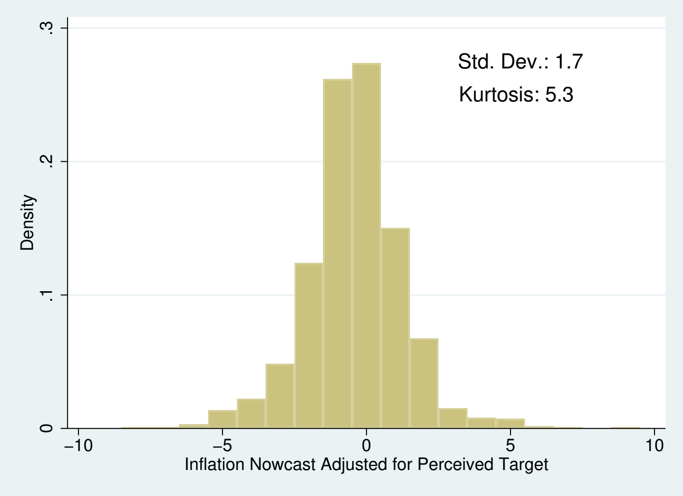
Motivation

- Information rigidities are important for transmission of monetary shocks.
- In the data, belief distributions have fat tails:
  - Firms are either very informed, or very uninformed.
- **This Paper:** A model of ex-ante identical firms that captures this.
- **Questions:**
  - Who drives monetary non-neutrality?
  - What are the relevant beliefs for monetary shocks?
  - What is a sufficient statistic for the real effects of monetary shocks?

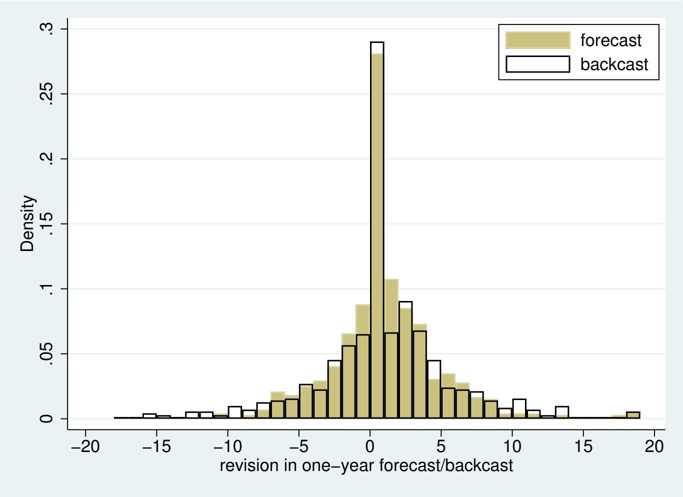
Motivation

Firms' nowcasts of inflation has a leptokurtic distribution.



Motivation

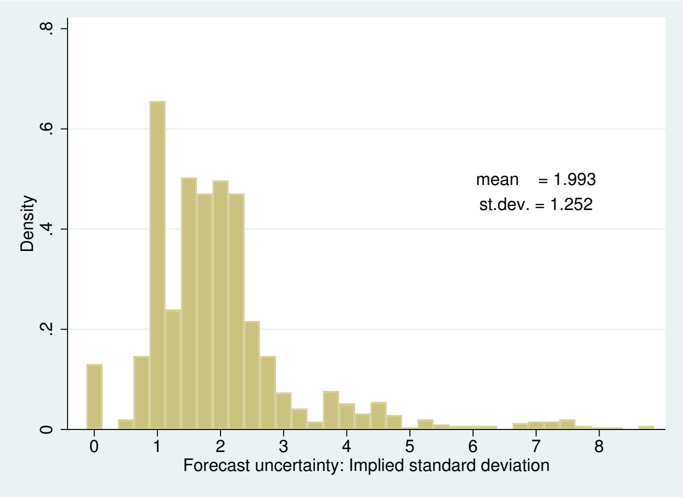
Distribution of belief revisions has fat tails.



Source: Kumar, Afrouzi, Coibion and Gorodnichenko (2015).

Motivation

There is a lot of heterogeneity in uncertainty across firms.



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Motivation

Firms who changed their prices more recently have more accurate expectations.

	Size of Nowcast Error			
	(1)	(2)	(3)	(4)
Price change (last 3m)	-1.42***		-1.25***	-0.89***
	(0.14)		(0.14)	(0.12)
Freq. of price reviews		-0.81***	-0.54***	0.10
		(0.08)	(0.08)	(0.08)
industry fixed effects	No	No	No	Yes
Observations	3,153	3,153	3,153	3,153

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Recap of evidence

- Firms are either very informed or very uninformed.
- When revising, they either don't revise or revise by a lot.
- There is a lot of heterogeneity in uncertainty.
- There is a positive correlation between being informed and having a recent price change.

Overview of Results

- Build a model of inattention + infrequent adjustments.
- Show evidence is consistent with random price adjustments:
  - firms don't acquire information in between price changes.
  - conditional on a price change they acquire a large amount of information.
  - There is selection in information acquisition.
- Derive sufficient statistics for monetary non-neutrality:
  - for announced shocks, the sufficient stat. comes from distribution of prices.
  - for unannounced shocks, the sufficient statistic comes from distribution of beliefs.

Literature

- Models of observation costs + menu costs and monetary non-neutrality
  - Reis(2006), Alvarez, Lippi, Paciello (2011, 2018). -> Perfect info conditional on observation.
- Models of consideration costs
  - Woodford(2009), Stevens(2015). -> Perfect info conditional on consideration.
- Models of inattention
  - Sims (2003,2006), Mackowiak and Wiederholt (2009, 2015) -> No nominal rigidity.

Outline

Outline

Model

Environment: Agents, Shocks and Payoffs.

- Time is continuous and indexed by $t \geq 0$.
- There is a measure of price-setting firms indexed by $i \in [0, 1]$.
- Each firm follows an exogenous *ideal* price:

$$dp^*_{i,t} = \mu dt + \sigma dW_{i,t}$$

- i's instantaneous loss from mispricing:

$$-B(p_{i,t} - p^*_{i,t})^2$$

Model

Environment: Information Structure and Cost of Attention.

- Firm i does not observe $p^*_{i,t}$ but see a signal process over time:

$$ds_{i,t} = p^*_{i,t}dt + \sigma_{s,i,t}dW_{s,i,t}$$

- Information sets:

$$S_{i,t} = \{s_{i,\tau} : 0 \leq \tau \leq t\} \cup S_{i,0}, \; S_{i,0} \text{ given.}$$

- Attention problem: firm chooses $\{\sigma_{s,i,t} \geq 0 : t \geq 0\}$.
- Instantaneous cost of attention: rate of reduction in differential entropy

$$\mathbb{I}(p^*_{i,t}|S_{i,t}) \equiv \lim_{\tau \downarrow 0} \frac{h(p^*_{i,t}|S_{i,t-\tau}) - h(p^*_{i,t}|S_{i,t})}{\tau}$$

Model

Environment: Frequency of Price Changes.

- Changing prices are costly.
- The opportunity of price change is a Poisson process:

$$dp_{i,t} = (\tilde{p}_{i,t} - p_{i,t})d\chi_{i,t}, \ \chi_{i,t} \sim \text{Poisson}(\theta_{i,t})$$

- Firm i chooses $\theta_{i,t}$ given a cost $c(\theta_{i,t})$.
- Micro-foundations: consideration costs as in Woodford (2009), Stevens (2018).
- $\theta_{i,t}$ can be state dependent:

Assumption

Firms cannot condition θ directly on their ideal price:

$$\theta_{i,t} \perp p^*_{i,t} | S_{i,t}.$$

Model

Environment: Firms' Problems.

$$\min_{\{\sigma_{s,i,t}\geq 0, \tilde{p}_{i,t}, \theta_{i,t}\geq 0 : t\geq 0\}} \mathbb{E}[\int_0^\infty e^{-\rho t}[\underbrace{B(p_{i,t}-p^*_{i,t})^2}_{\text{loss from mis-pricing}} + \underbrace{\psi\mathbb{I}(p^*_{i,t}|S_{i,t})}_{\text{cost of information}} + \underbrace{c(\theta_{i,t})}_{\text{cost of consideration}}]dt|S_{i,0}]$$

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Lemma

Suppose $c(\theta) = \Xi\bar{\theta}^{1-\gamma}\theta^{\gamma}$. Then,

- *As $\gamma \to \infty$: $\theta_{i,t} \to \bar{\theta} \Rightarrow$ Calvo.*
- *When $\gamma = 1$: $c(\theta_{i,t}) = \Xi 1\{dp_{i,t} \neq 0\} \Rightarrow$ Menu Cost.*

Model

Characterization of Firms' Problem: Evolution of Beliefs

Lemma

Given $S_{i,0}$ and a sequence $\{\sigma_{s,i,t} \geq 0 : t \geq 0\}$, the firm's conditional beliefs $p^*_{i,t}|S_{i,t} \sim \mathcal{N}(\hat{p}_{i,t}, z_{i,t})$ evolve according to

$$d\hat{p}_{i,t} = \lambda_{i,t}(p^*_{i,t} - \hat{p}_{i,t})dt + \sqrt{\lambda_{i,t} z_{i,t}} dW_{s,i,t} \qquad \text{(evolution of the mean)}$$

$$dz_{i,t} = (\sigma^2 - \lambda_{i,t} z_{i,t})dt \qquad \text{(evolution of the variance)}$$

$$\hat{p}_{i,0}, z_{i,0} \text{ given.}$$

where $\lambda_{i,t} \equiv z_{i,t}/\sigma^2_{s,i,t}$ is the Kalman-Bucy gain of i at t.

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Lemma

The rate of reduction in differential entropy is the Kalman-Bucy gain:

$$\mathbb{I}(p^*_{i,t}|S_{i,t}) = \lambda_{i,t}.$$

Model

Characterization of Firms' Problem: Gaps

Definition

We define firm i's ***true price gap****,* ***perceived price gap****, and* ***belief gap*** *at time t as*

$$x^*_{i,t} \equiv p^*_{i,t} - p_{i,t}, \quad x_{i,t} \equiv \mathbb{E}[p^*_{i,t}|S_{i,t}] - p_{i,t}, \quad b_{i,t} \equiv p^*_{i,t} - \mathbb{E}[p^*_{i,t}|S_{i,t}],$$

respectively.

- $x^*_{i,t}$ determines firm's loss from mispricing.
- $b_{i,t}|S_{i,t} \sim \mathcal{N}(0, z_{i,t})$ captures imperfect information.
- $x_{i,t}$ captures nominal rigidity.

Model

Characterization of Firms' Problem: HJBs

$$\underbrace{\mathbb{E}[{x^*_{i,t}}^2 | S_{i,t}]}_{\text{perceived loss}} = \underbrace{x^2_{i,t}}_{\text{nominal rigidity}} + \underbrace{z_{i,t}}_{\text{[subjective] uncertainty}}$$

Lemma

The firms' problem is characterized by state variables x and z through

$$\begin{aligned}
\rho\ell(x,z) &= B(x^2+z) + \sigma^2\partial_z\ell(x,z) + \mu\partial_x\ell(x,z) \\
&\quad + \min_{\theta\geq 0}\{\theta[\ell(\tilde{x},0) - \ell(x,z)] + c(\theta)\} \\
&\quad + \min_{\lambda\geq 0}\left\{[\frac{1}{2}\partial_{xx}\ell(x,z) - \partial_z\ell(x,z)]\lambda z + \psi\lambda\right\}, \\
\tilde{x} &\equiv \arg\min_x l(x,z) \\
\partial_z l(\tilde{x},z)z &\leq \psi
\end{aligned}$$

Outline

1. Model
2. Results
3. Aggregation
4. Implications for Monetary Non-Neutrality in Calvo

Model

Characterization of Firms' Problem: Optimal Attention

Proposition

There exists a baseline uncertainty, Z^*, such that

- *if $z < Z^*$, the firm acquires no information.*

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- *if $z \geq Z^*$, if the firm acquires information, it acquires enough to reset its posterior uncertainty to Z^*.*

$$Z^* \approx \sigma\sqrt{\frac{\psi}{B}} + \frac{\rho\psi}{B}$$

- Cost of processing information is linear in "amount" of information.
- If information is acquired gradually for future, the agent is better off to wait and buy it all together in the future.

Proposition

- *In the Calvo extreme, firms never acquire information in between price changes.*
- *In the menu cost extreme, firms constantly acquire information in their inaction region to maintain* Z^*.

- Calvo: information is only used for estimating the size of price change. Why acquire when opportunity has not arrived?

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- Menu cost: in addition to estimating the size of price change, information is also used for determining when to change.

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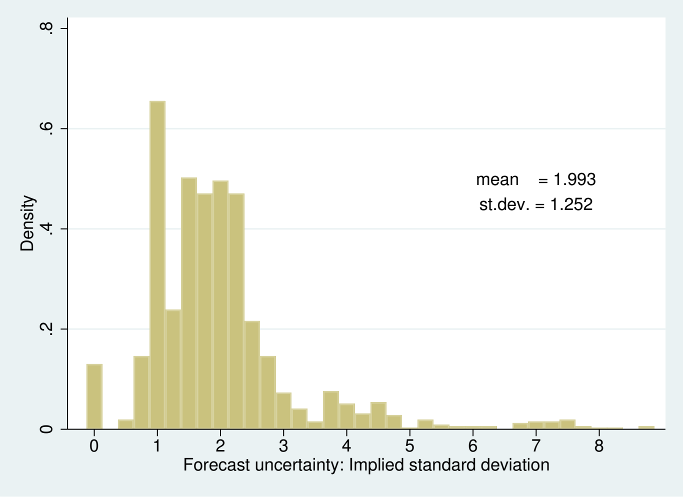
- Calvo: information is only used for estimating the size of price change. Why acquire when opportunity has not arrived?
- Menu cost: in addition to estimating the size of price change, information is also used for determining when to change.
- The cost of Type I and Type II errors are so large that keeps firms always on their toes.

Outline

Aggregation Results for:

- distribution of uncertainty.
- distribution of belief revisions.
- distribution of true price gaps to study real effects of monetary policy.

Aggregation
Distribution of Uncertainty



Source: Kumar, Afrouzi, Coibion and Gorodnichenko (2015).

Aggregation

Distribution of Uncertainty

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The time invariant distribution of uncertainty

- *in the menu cost model is a univariate degenerate distribution at Z^*.*

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The time invariant distribution of uncertainty

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- *in the Calvo model is an exponential with rate θ/σ^2 shifted by Z^*.*

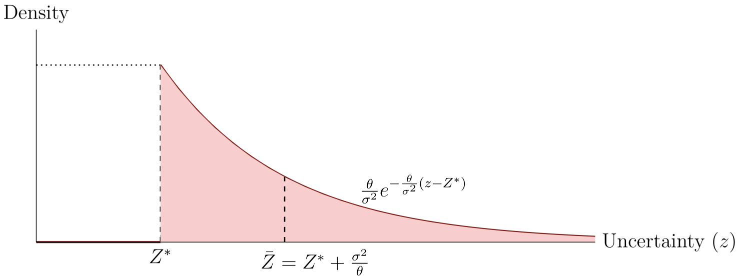
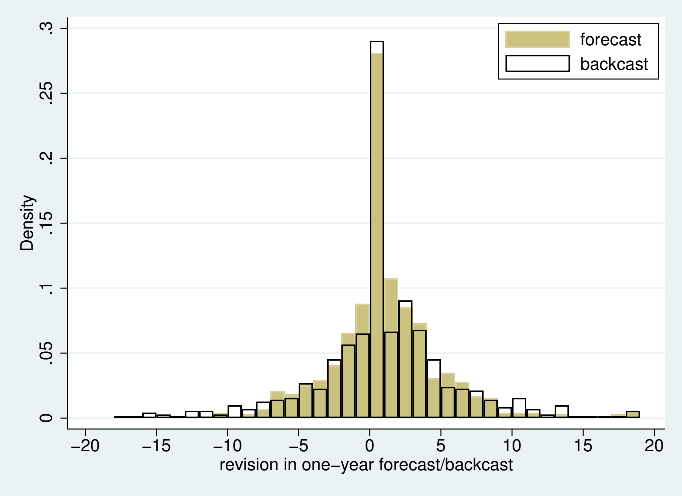


Figure I: Distribution of Uncertainty Across Firms

Aggregation

Distribution of Belief Revisions



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Aggregation

Distribution of Belief Revisions

Proposition

The time invariant distribution of belief revisions

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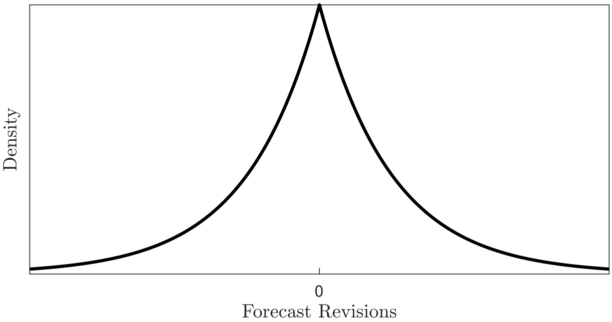
Aggregation

Distribution of Belief Revisions

Proposition

The time invariant distribution of belief revisions

- *in the menu cost model is* $\mathcal{N}(0, \sigma^2)$.
- *in the Calvo model is a Laplace with scale* $\sqrt{2\theta}/\sigma$.



Aggregation
Steady State in Calvo

- Let $\tilde{F}$ be the invariant (steady state) joint distribution of (b, x, z) in the Calvo model.
- We want to understand the effect of shocks to each element, so its important to know the steady state joint distribution.

Proposition

In the Calvo model, the steady state joint distribution of (b, x, z) is such that

$$\tilde{F}(b|x, z) = \tilde{F}(b|z) = \mathcal{N}(0, z)$$

with the marginals of x and z being exponential distributions.

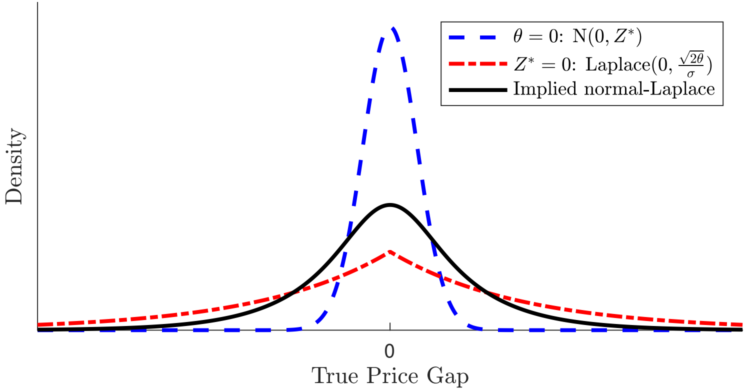
Aggregation

Distribution of belief gaps

Proposition

In the Calvo model, the time-invariant distribution of belief gaps is a normal-Laplace distribution; it is the distribution of X where

$$X = X_n + X_L,$$
$$X_L \perp X_n$$
$$Y_n \sim \mathcal{N}(0, Z^*),$$
$$X_L \sim Laplace(\frac{\mu}{\theta}, \frac{\sqrt{2\theta}}{\sigma}, \sqrt{1 + \frac{\mu^2}{2\theta\sigma^2}} - \sqrt{\frac{\mu^2}{2\theta\sigma^2}}).$$


$\theta = 0$: N$(0, Z^*)$
$Z^* = 0$: Laplace$(0, \frac{\sqrt{2\theta}}{\sigma})$
Implied normal-Laplace
Density
0
True Price Gap

Aggregation

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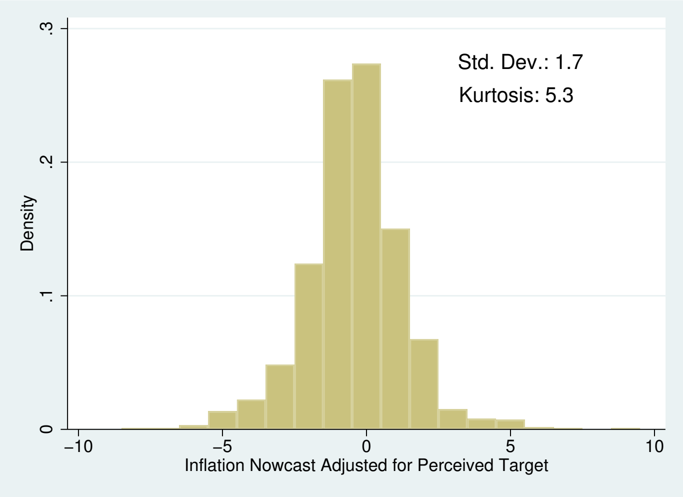

Outline

Monetary Non-Neutrality with Calvo

- Let $y_{i,t} \equiv x^*_{i,t} = x_{i,t} + b_{i,t}$.
- Given the initial belief and perceived gap of firm define:

$$Y(x,b,z) \equiv \mathbb{E}_0[\int_0^\infty y_{i,t} dt | x_{i,0} = x, b_{i,0} = b, z_{i,0} = z].$$

Lemma

$$Y(x,b,z) = \theta^{-1} x + m(z) b, \; m'(z) < 0$$

Definition

Given an initial distribution for $(x_{i,0}, b_{i,0}, z_{i,0})_{i \in [0,1]} \sim F(b,x,z)$, the cumulative response of output is

$$\mathcal{M}(F) = \int Y(b,x,z) dF(b,x,z)$$

Monetary Non-Neutrality

- Two types of unanticipated monetary shocks:
  - unanticipated shock to perceived price gaps.
  - unanticipated shock to belief gaps.

Proposition

Let $\tilde{F}$ be the time-invariant distribution of (x, b, z) in the model. Then,

- $\mathcal{M}(\tilde{F}) = 0$.
- *Let $F_b = \tilde{F}(x, b-1, z)$ be a shock of size 1 to z. Then,*

$$\mathcal{M}(F_b) = \frac{\bar{Z}}{\sigma^2}$$

- *Let $F_x = \tilde{F}(x-1, b, z)$ be a shock of size 1 to x. Then,*

$$\mathcal{M}(F_x) = \frac{1}{\theta}$$

Monetary Non-Neutrality

Why the difference?

- Because it takes time for firms to become aware of the shock when it is unannounced:

$$db = -\lambda(z)b + U,$$
$$\lambda(z) = 1 - \frac{Z^*}{z}$$

- In fact:

$$\mathcal{M}(F_b) - \mathcal{M}(F_x) = \frac{Z^*}{\sigma^2}$$

- Need to know uncertainty conditional on price change.

Monetary Non-Neutrality

Identifying uncertainty

- Can we still identify non-neutrality of money from distribution of price changes?

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Proposition

The distribution of price changes is invariant to Z^*.

Intuition of Proof: take an arbitrary price change,

$$\Delta p_{i,t} = \lambda_{i,t}(p^*_{i,t} + noise - p_{i,t-h}) \tag{1}$$

- Optimality of $\lambda_{i,t}$ implies $var(\Delta p_{i,t}) = \sigma^2 h$.

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- In hypothetical economy assign $\Delta p_{i,t}$ to a firm whose ideal price is $p_{i,t}$.
- The hypothetical economy is as if it has no information friction.
- But it has the same distribution of price changes.

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Conclusion

- Built a model to study interaction of sticky prices and non-neutrality.
- Showed there is selection in information acquisition conditional on price change.
- Derived a sufficient statistic for non-neutrality of money under inattention.