Dynamic Rational Inattention and the Phillips Curve

Hassan Afrouzi
Columbia

Choongryul Yang
UT Austin

Motivation

- Evidence on belief formation suggests information acquisition is costly:
  - households and firms ignore available information
  - forecast errors are correlated with forecast revisions

Motivation

- Evidence on belief formation suggests information acquisition is costly:
  - households and firms ignore available information
  - forecast errors are correlated with forecast revisions

- Dynamic Rational Inattention:
  - a disciplined model of costly information acquisition
    (one parameter + cost function)
  - payoffs determine incentives for attention
  - generate inertia, persistence, hump-shaped responses etc.

Motivation

- Evidence on belief formation suggests information acquisition is costly:
  - households and firms ignore available information
  - forecast errors are correlated with forecast revisions

- Dynamic Rational Inattention:
  - a disciplined model of costly information acquisition
    (one parameter + cost function)
  - payoffs determine incentives for attention
  - generate inertia, persistence, hump-shaped responses etc.

- Challenges:
  - notoriously complex and slow to solve (endogenously binding constraints)
  - technological barriers to test, or integrate with different shocks and frictions

What do we do?

- **Methods**: solve Dynamic Rational Inattention Problems (DRIPs)
  - fully characterize the transition dynamics of attention in LQG settings

What do we do?

- **Methods**: solve Dynamic Rational Inattention Problems (DRIPs)
    - fully characterize the transition dynamics of attention in LQG settings
    - dynamics are governed by inaction regions for uncertainty

What do we do?

- **Methods**: solve Dynamic Rational Inattention Problems (DRIPs)
  - fully characterize the transition dynamics of attention in LQG settings
  - dynamics are governed by inaction regions for uncertainty
  - provide a fast, general and robust method for obtaining the solution

What do we do?

- **Methods**: solve Dynamic Rational Inattention Problems (DRIPs)
  - fully characterize the transition dynamics of attention in LQG settings
  - dynamics are governed by inaction regions for uncertainty
  - provide a fast, general and robust method for obtaining the solution

- **Application**: an attention-driven theory of the Phillips curve
  - when MP is more hawkish the RI Phillips curve is flatter
  - when MP becomes more dovish (or at ZLB) the RI Phillips curve is
    - **completely flat** in the **short-run**
    - **steeper** in the **long-run**
  - inflation expectations are more anchored when MP is more hawkish

What do we do?

- **Methods**: solve Dynamic Rational Inattention Problems (DRIPs)
    - fully characterize the transition dynamics of attention in LQG settings
    - dynamics are governed by inaction regions for uncertainty
    - provide a fast, general and robust method for obtaining the solution

- **Application**: an attention-driven theory of the Phillips curve
    - when MP is more hawkish the RI Phillips curve is flatter
    - when MP becomes more dovish (or at ZLB) the RI Phillips curve is
        - **completely flat** in the **short-run**
        - **steeper** in the **long-run**
    - inflation expectations are more anchored when MP is more hawkish

- **Quantitative:** is the theory quantitatively relevant?
    - calibrate a 3-equation general equilibrium RI model to post-Volcker data
    - model matches pre-Volcker inflation/output volatility as nontargeted moments
    - estimated slope of the Phillips curve declines by up to 75% in simulated data

Literature Review

- Rational Inattention in Macro: Sims (2003); Peng and Xiong (2006); Luo (2008); Mackowiack and Wiederholt (2009); Woodford (2009); Modria and Wu (2010); Paciello (2012); Lue et al. (2012); Tutino (2013); Paciello and Wiederholt (2014); Stevens (2015); Mackowiack and Wiederholt (2015); Pasten and Schoenle (2016); Zorn (2016); Afrouzi (2017); Ilut and Valchev (2017); Khaw and Zorrilla (2019); Yang (2019).
- Methods on Solving LQG Dynamic Rational Inattention Models: Sims (2003, 2010); Mackowiack, Matejka and Wiederholt (2018); Fulton (2018); Miao, Wu and Young (2020).
- Change in the Slope of Phillips Curve: Coibion and Gorodnichenko (2015); Blanchard (2016); Bullard (2018); Hooper, Mishkin and Sufi (2019); Del Negro, Lenza, Primiceri and Tambalotti (2020).
- Imperfect Information and Phillips Curve: Lucas, 1982; Mankiw ad Reis, 2002; Woodford, 2003; Nimark, 2008; Angeletos and La'O, 2009; Angeletos and Huo, 2018; Angeletos and Lian, 2018.
- Alternative explanations:
  - Identification of PC: McLeany and Tenreyro (2019).
  - Convexities in Phillips curve: Kumar and Orrenius (2016); Babb and Detmeister (2017).

- Quick Overview of Dynamic Rational Inattention Problems (DRIPs)

- A Simple GE Model with an Attention Driven Phillips curve

- A Calibrated 3-Equation Rational Inattention Model

$$v(\vec{a}_t, \vec{x}_t) = -\frac{1}{2}(\vec{a}_t' - \vec{x}_t'\mathsf{H})(\vec{a}_t - \mathsf{H}'\vec{x}_t), \quad \vec{x}_t = A\vec{x}_{t-1} + Q\vec{u}_t$$

$$v(\vec{a}_t, \vec{x}_t) = -\frac{1}{2}(\vec{a}_t' - \vec{x}_t'H)(\vec{a}_t - H'\vec{x}_t), \quad \vec{x}_t = A\vec{x}_{t-1} + Q\vec{u}_t$$

- End product of Rational Inattention: a joint process for $(\vec{x}_t, \hat{x}_t, \vec{a}_t)$ where

$$\hat{x}_t \equiv \mathbb{E}[\vec{x}_t|S^t] \Rightarrow \vec{a}_t = H'\hat{x}_t$$

- With full information:

$$\hat{x}_t = \vec{x}_t, \quad \vec{a}_t = H'\vec{x}_t$$

- With LQG, beliefs are Gaussian: $\vec{x}_t | S^t \sim N(\hat{x}_t, \Sigma_{t|t})$
- Theorem 1: derives an Euler equation for characterizing $(\Sigma_{t|t})_{t=0}^{\infty}$

DRIPs with LQG

- With LQG, beliefs are Gaussian: $\vec{x}_t | S^t \sim N(\hat{x}_t, \Sigma_{t|t})$
- Theorem 1: derives an Euler equation for characterizing $(\Sigma_{t|t})_{t=0}^{\infty}$
- Gaussian beliefs are generated by Gaussian signals + Kalman filtering

$$S^t = \{Y'_\tau \vec{x}_\tau + \vec{z}_\tau\}_{\tau=0}^t \cup S^{-1}$$

- Theorem 2: characterize the sequence $(Y_t)_{t=0}^{\infty}$
  - Rank of Y_t determines the number of signals (sparsity)
  - Eigenvalues of Y_t determine how inertial beliefs are (signal-to-noise ratios)
  - Eigenvectors of Y_t determine how information is garbled (information spillovers)

DRIPs with LQG

- With LQG, beliefs are Gaussian: $\vec{x}_t | S^t \sim N(\hat{x}_t, \Sigma_{t|t})$
- Theorem 1: derives an Euler equation for characterizing $(\Sigma_{t|t})_{t=0}^{\infty}$
- Gaussian beliefs are generated by Gaussian signals + Kalman filtering

$$S^t = \{Y'_\tau \vec{x}_\tau + \vec{z}_\tau\}_{\tau=0}^t \cup S^{-1}$$

- Theorem 2: characterize the sequence $(Y_t)_{t=0}^{\infty}$
  - Rank of Y_t determines the number of signals (sparsity)
  - Eigenvalues of Y_t determine how inertial beliefs are (signal-to-noise ratios)
  - Eigenvectors of Y_t determine how information is garbled (information spillovers)
- On the web:
  - A software package that solves and returns the solution, IRFs and simulated paths of actions/beliefs that is general and fast enough for quantitative work.
    ▸ GitHub   ▸ Pricing Example   ▸ Mackowiak and Wiederholt (2009)   ▸ Sims (2010)
    ▸ Mackowiak, Matejka and Wiederholt (2018)   ▸ Quantitative Model

A Simple GE Model with Rational Inattention

- Fully attentive household:

$$\max_{\{(C_{i,t})_{i\in[0,1]},N_t\}_{t=0}^{\infty}} \mathbb{E}_0^f[\sum_{t=0}^{\infty} \beta^t(\log(C_t) - N_t)]$$

$$s.t. \int_0^1 P_{i,t}C_{i,t}di + B_t \leq W_tN_t + (1 + i_{t-1})B_{t-1} + T_t$$

$$C_t = \left[\int_0^1 C_{i,t}^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}$$

- Aggregate wage: $W_t = Q_t$ where $Q_t = P_tC_t$.

Households

- Log-linearized Euler equation:

$$c_t = \mathbb{E}_t^f[c_{t+1}] - (i_t - \mathbb{E}_t^f[\pi_{t+1}])$$

  where small letters denote logs.
- Monetary policy responds to inflation and output growth:

$$i_t = \phi\pi_t + \phi\Delta y_t - \sigma_u u_t, \ u_t \sim \mathcal{N}(0,1)$$

Lemma

Suppose $\phi > 1$. Then, nominal demand is a random walk:

$$q_t = q_{t-1} + \frac{\sigma_u}{\phi}u_t.$$

Firms

- Rationally inattentive.
- Produce goods and information capacity with linear tech. in labor:
- Set prices and satisfy implied demand.

$$\max_{(\kappa_{i,t}, S_{i,t}, P_{i,t})_{t \geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{Q_t}(P_{i,t}Y_{i,t} - W_t N_{i,t}^p - W_t N_{i,t}^d)|S_i^{-1}\right]$$

$$s.t. \qquad Y_{i,t} = N_{i,t}^p = C_t P_t^\theta P_{i,t}^{-\theta} \qquad \text{(production tech. and demand)}$$

Firms

- Rationally inattentive.
- Produce goods and information capacity with linear tech. in labor:
- Set prices and satisfy implied demand.

$$\max_{(\kappa_{i,t}, S_{i,t}, P_{i,t})_{t \geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{Q_t}(P_{i,t}Y_{i,t} - W_t N_{i,t}^p - W_t N_{i,t}^d)|S_i^{-1}\right]$$

$$s.t. \quad Y_{i,t} = N_{i,t}^p = C_t P_t^\theta P_{i,t}^{-\theta} \qquad \text{(production tech. and demand)}$$

$$\mathbb{I}(S_i^t; X^t | S_i^{t-1}) \leq \omega^{-1} N_{i,t}^d \qquad \text{(information flow constraint)}$$

Firms

- Rationally inattentive.
- Produce goods and information capacity with linear tech. in labor:
- Set prices and satisfy implied demand.

$$\max_{(\kappa_{i,t}, S_{i,t}, P_{i,t})_{t \geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{Q_t}(P_{i,t}Y_{i,t} - W_t N_{i,t}^p - W_t N_{i,t}^d)|S_i^{-1}\right]$$

$$s.t. \quad Y_{i,t} = N_{i,t}^p = C_t P_t^{\theta} P_{i,t}^{-\theta} \qquad \text{(production tech. and demand)}$$

$$\mathbb{I}(S_i^t; X^t | S_i^{t-1}) \leq \omega^{-1} N_{i,t}^d \qquad \text{(information flow constraint)}$$

$$S_i^t = S_i^{t-1} \cup S_{i,t} \qquad \text{(evolution of information stock)}$$

- A second order approximation to firms' problem:

$$V_0(var(q_0|S^{-1})) = \min_{\{p_{i,t}, S_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [(\theta - 1)(p_{i,t} - q_t)^2 + \omega \mathbb{I}(S_i^t, q^t|S_i^{t-1})|S_i^{-1}]$$

- Given a sequence of choices for capacities:

$$p_{i,t} = \mathbb{E}[q_t|S_i^t] \qquad \text{(prices track marginal costs)}$$
$$S_{i,t} = q_t + e_{i,t} \qquad \text{(information flow)}$$

where signal to noise ratio is such that

$$\mathbb{E}[q_t|S_i^t] = (1 - \kappa_{i,t})\mathbb{E}[q_{t-1}|S_i^{t-1}] + \kappa_{i,t}S_{i,t}$$

- Since firms do not observe q_t perfectly, $\sigma_t^2 \equiv var(q_t | S^{t-1})$ is a state variable.
- Firms' problem reduces to choosing a Kalman gain in $[0, 1]$:

$$v(\sigma_t^2) = \min_{\kappa_t \in [0,1]} \{(\theta - 1)(1 - \kappa_t)\sigma_t^2 - \omega \log(1 - \kappa_t) + \beta v(\sigma_{t+1}^2)\}$$

$$s.t. \quad \sigma_{t+1}^2 = (1 - \kappa_t)\sigma_t^2 + \frac{\sigma_u^2}{\phi^2}, \quad \sigma_0^2 \text{ given.}$$

- Optimal κ_t depends on ϕ only through dynamics ($\beta > 0$).

Firms

Proposition

Firms only pay attention to the monetary policy shocks if their prior uncertainty is outside of an attention inaction region.

Proposition

Firms only pay attention to the monetary policy shocks if their prior uncertainty is outside of an attention inaction region. Formally, the policy function of a firm for choosing their posterior uncertainty is

$$\kappa_t = \begin{cases} 0 & \sigma_t^2 \leq \underline{\sigma}^2 \\ \bar{\kappa} & \sigma_t^2 > \underline{\sigma}^2 \end{cases}$$

where if $\beta \to 1$ and $\sigma_u^2 \ll \omega$:

$$\underline{\sigma}^2 \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\omega}{\theta - 1}}, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\theta - 1}{\omega}}$$

Firms

Proposition

Suppose all firms start from the same prior uncertainty. Then, the Phillips curve of this economy is

$$\pi_t = \frac{\kappa_t}{1 - \kappa_t} y_t$$

$$= \begin{cases} 0 & \sigma_t^2 \leq \underline{\sigma}^2 \\ \frac{\bar{\kappa}}{1 - \bar{\kappa}} y_t & \sigma_t^2 \geq \underline{\sigma}^2 \end{cases}$$

$$\underline{\sigma}^2 \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\omega}{\theta - 1}}, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\theta - 1}{\omega}}$$

$$\underline{\sigma}^2 \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\omega}{\theta - 1}}, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\theta - 1}{\omega}}$$

Corollary
Suppose the economy is in the steady state of its attention problem, and consider an unexpected decrease in $\frac{\sigma_u}{\phi}$. Then,

$$\underline{\sigma}^2 \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\omega}{\theta - 1}}, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\theta - 1}{\omega}}$$

Corollary
Suppose the economy is in the steady state of its attention problem, and consider an unexpected decrease in $\frac{\sigma_u}{\phi}$. Then,

1. *the inaction region tightens and the economy immediately jumps to a new steady state of the attention problem.*
2. *the Phillips curve is flatter.*

$$\underline{\sigma}^2 \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\omega}{\theta - 1}}, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\theta - 1}{\omega}}$$

Corollary

Suppose the economy is in the steady state of its attention problem, and consider an unexpected increase in $\frac{\sigma_u}{\phi}$. Then,

1. *Inaction region widens and the Phillips curve becomes temporarily flat until firms' uncertainty exit this region (Short-run $\approx \frac{\Delta \phi}{\sigma_u} \sqrt{\frac{\omega}{\theta - 1}}$)*

$$\underline{\sigma}^2 \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\omega}{\theta - 1}}, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi} \sqrt{\frac{\theta - 1}{\omega}}$$

Corollary

Suppose the economy is in the steady state of its attention problem, and consider an unexpected increase in $\frac{\sigma_u}{\phi}$. Then,

1. *Inaction region widens and the Phillips curve becomes temporarily flat until firms' uncertainty exit this region (Short-run $\approx \frac{\Delta\phi}{\sigma_u} \sqrt{\frac{\omega}{\theta-1}}$)*

2. *Once firms' uncertainty exits inaction region, the economy enters its new steady state in which the Phillips curve is steeper.*

Proposition

Let $\hat{\pi}_t \equiv \int_0^1 \mathbb{E}_{i,t}[\pi_t]di$ denote the average expectation of firms about aggregate inflation at time t. Then, in the steady state of the attention problem,

- *the relationship between inflation expectations, $\hat{\pi}_t$, and output gap, y_t, is given by*

$$\hat{\pi}_t = (1 - \bar{\kappa})\hat{\pi}_{t-1} + \frac{\bar{\kappa}^2}{(2 - \bar{\kappa})(1 - \bar{\kappa})}y_t, \quad \bar{\kappa} \approx \frac{\sigma_u}{\phi}\sqrt{\frac{\theta - 1}{\omega}}$$

$\Rightarrow$ *inflation expectations are more anchored when policy is more hawkish.*

▸ *IRFs*

Quantitative Analysis

- So far we assumed $p_t^* = q_t$. General case: $p_t^* = p_t + \alpha \tilde{y}_t$

- So far we assumed $p_t^* = q_t$. General case: $p_t^* = p_t + \alpha \tilde{y}_t$
- 3-Equation RI model with TFP and MP shocks:

$$\tilde{y}_t = \mathbb{E}_t^f \left[\tilde{y}_{t+1} - \frac{1}{\sigma} \left(i_t - \pi_{t+1} - r_t^n \right) \right] \qquad \text{(Euler equation)}$$

$$p_{i,t} = \mathbb{E}_{i,t} \left[p_t + \alpha \tilde{y}_t \right], \forall i \in [0,1] \qquad \text{(Pricing equation)}$$

$$i_t = \rho i_{t-1} + (1 - \rho) \left(\phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + \phi_{\Delta y} \Delta y_t \right) + u_t \qquad \text{(Taylor rule)}$$

- Solution requires solving DRIP under the fixed point for state-space rep. of p_t^*

- Calibrate the model to Post-Volcker, Pre-ZLB US data.
- Replace the Taylor rule with Pre-Volcker and check if the model can explain the volatility of output gap and inflation

  (à la Clarida *et al.*, 2000; Mackowiak and Wiederholt, 2015)   ▸ Estimates   ▸ Calibration

	Pre-Volcker (non-targeted)		Post-Volcker (targeted)	
	Data	Model	Data	Model
Standard deviation of inflation	0.025	0.025	0.015	0.015
Standard deviation of output	0.020	0.022	0.018	0.018
Correlation of inflation and output	0.245	0.242	0.209	0.209

What Does the Model Say about the Slope of the PC?

- We cannot estimate our PC for Pre-Volcker (missing data).
- However, we can estimate the NKPC with simulated data from model.

$$\pi_t = \alpha + \gamma E_t \pi_{t+1} + (1 - \gamma)\pi_{t-1} + \kappa x_t + \varepsilon_t$$

	(1) Output gap		(2) Output		(3) Adj. output gap	
	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker
Slope of NKPC (κ)	1.160	0.304	0.035	0.027	0.024	-0.012
	(0.029)	(0.007)	(0.001)	(0.001)	(0.007)	(0.003)
Forward-looking (γ)	0.666	0.612	0.549	0.499	0.554	0.512
	(0.005)	(0.003)	(0.002)	(0.001)	(0.002)	(0.001)

Conclusion

- Solve DRIPs and propose an attention driven theory of the Phillips curve.

- The slope of the PC is endogenous to monetary policy.
  - Firms stop paying attention to MP shocks when MP commits more to stabilizing nominal variables.
  - The PC becomes flatter and less forward-looking.

- Unanticipated shocks to how MP is conducted has asymmetric effects:
  - A more hawkish policy immediately moves the economy to a flatter Phillips curve.
  - A more dovish policy makes the PC flat in the short-run and steeper (and more forward-looking) in the long-run.

- A calibrated model explains the decline in the slope of the PC.

Appendix

	constant	ρ	ϕ_π	$\phi_{\Delta y}$	ϕ_x
Pre-Volcker	0.096	0.957	1.589	1.028	1.167
(1969–1978)	(0.187)	(0.022)	(0.847)	(0.601)	(0.544)
Post-Volcker	-0.310	0.961	2.028	3.122	0.673
(1983–2007)	(0.062)	(0.015)	(0.617)	(1.090)	(0.234)

Notes: This table reports least squares estimates of the Taylor rule. We use the Greenbook forecasts of current and future macroeconomic variables. The interest rate is the target federal funds rate set at each meeting from the Fed. The measure of the output gap is based on Greenbook forecasts. We consider two time samples: 1969–1978 and 1983–2002. Newey-West standard errors are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.
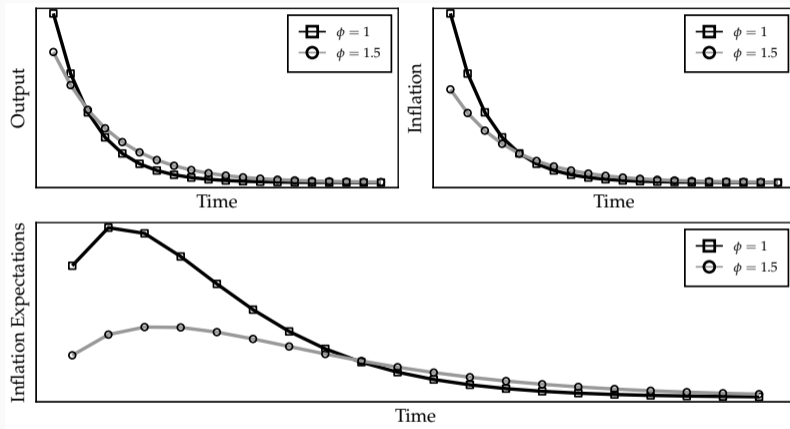
Figure 1: Impulse Responses to a 1 Std. Dev. Expansionary Monetary Policy Shock

Calibration Summary

Parameter	Value	Moment Matched / Source
Panel A. Calibrated parameters		
Information cost (ω)	0.70×10^{-3}	Cov. matrix of GDP and inflation
Persistence of productivity shocks (ρ_a)	0.850	Cov. matrix of GDP and inflation
S.D. of productivity shocks (σ_a)	1.56×10^{-2}	Cov. matrix of GDP and inflation
Panel B. Assigned parameters		
Time discount factor (β)	0.99	
Elasticity of substitution across firms (θ)	10	Firms' average markup
Elasticity of intertemporal substitution ($1/\sigma$)	0.4	Aruoba et al. (2017)
Inverse of Frisch elasticity (ψ)	2.5	Aruoba et al. (2017)
S.D. of monetary shocks (σ_u)	0.28×10^{-2}	Romer and Romer (2004)
Panel C. Counterfactual model parameters (Pre-Volcker: 1969–1978)		
S.D. of monetary shocks (σ_u)	0.54×10^{-2}	Romer and Romer (2004)